Wiener Filter Design in Power-Quality Improvement

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***Abstract*— This paper designs a finite impulse response (FIR) filter using the Wiener-Hopff Equation, to mitigate power-quality (PQ) disturbances caused by harmonics and flickers. The filter design is based on retrieving the the fundamental component and suppressing all other disturbances, or alternatively the filter is designed based on extracting the disturbances first and then removing them from the original signal. Both approaches are investigated and the advantages and disadvantages of each scheme is examined using frequency response analysis. Furthermore, the effect of filter order on the overall performance of the system is considered.**

***Index Terms*— Finite impulse response (FIR), adaptive filter (AF), mean square error (MSE), and power-quality (PQ).**

I. Introduction

SOLID-STATE control of ac power using thyristors and other semiconductor switches is widely employed to feed controlled electric power to electrical loads, such as adjustable speed drives (ASDs), computer power supplies, etc. As nonlinear loads, these solid-state converters draw harmonic and reactive power components of current from ac mains. In three-phase systems, they could also cause unbalance and draw excessive neutral currents. The injected harmonics, reactive power burden, unbalance, and excessive neutral currents cause low system efficiency and poor power factor. They also cause disturbance to other consumers and interference in nearby communication networks. Therefore, power-quality (PQ) problems have become the focus of interest, resulting in a new wave of research seeking effective and efficient schemes for suppressing the effect of disturbances.

Extensive surveys [1], [2] have been carried out to quantify the problems associated with electric power networks having nonlinear loads. Conventionally passive LC filters were used to reduce harmonics and capacitors were employed to improve the power factor of the ac loads. However, passive filters have the demerits of fixed compensation, large size, and resonance. The increased severity of harmonic pollution in power networks has attracted the attention of power electronics and power system engineers to develop dynamic and adjustable solutions to the power quality problems using adaptive filters (AF), [3].

[4]–[6] represents much of the work done in the area of mitigating of the voltage quality problems, (voltage sags, swells, and fluctuations), using different time-domain and frequency-domain techniques. The proposed scheme in [6] seeks to extract the flicker disturbances, however the algorithm is too complicated for practical implementation. The Kalman filter and the least absolute value schemes have been proposed in [7] and [8] require a state-space model to be able to extract the disturbances, nevertheless the proposed approach increases the mathematical complexity and hinders the practical implementation.

In this paper we investigate the use Wiener filters to improve the PQ for signals suffering from harmonics and flickers [3]. Two different approaches are taken into consideration where in the first scheme the designed filter seeks to extract the fundamental frequency or the desired signal and the second scheme where the filter is designed to extract the nuisance parameters first and subsequently the effect of flickers and harmonics are removed from the original signal. The robustness of the two algorithms are compared and investigated using frequency response analysis. Moreover, this paper investigates the effect filter order on the overall performance of the algorithm in terms of improved PQ.

This paper is organized as follows: Section II outlines the system model and establishes the algorithms under consideration to determine the optimum filter coefficients with respect to a minimum mean square (MSE) design criteria. Section III discusses the extensive simulation results and examines both filter design algorithms in terms of their effectiveness in removing the disturbances.

This following notation is used throughout this report: italic letters (x) represent scalar quantities, bold lower case letters (**x**) represent vectors, bold upper case letters (**X**) represent matrices, and $(.)^T$ denotes transpose.

II. System and Filter Model

In this section we define the system model for the proposed project. Eq. (1) defines the relationship between the received signal $r(t)$ and the desired signal $d(t)$.

$$r(t) = d(t) + h(t) + f(t), \tag{1}$$

where $h(t)$ represents the harmonics and $f(t)$ represents the flicker effecting $d(t)$. In this project $d(t)$ is defined as

$$d(t) = 130\sin(2\pi 60t), \tag{2}$$

$h(t)$ is represented as

$$f(t) = a_{h50}\sin(2\pi 50t) + a_{h70}\sin(2\pi 70t), \tag{3}$$

where a_{h50} and a_{h70} are the amplitude of flickers at 50 and 70 Hz, respectively, and $f(t)$ is defined as

$$\begin{aligned} h(t) = a_{f180}\sin(2\pi 180t) + a_{f300}\sin(2\pi 300t) \\ + a_{f420}\sin(2\pi 420t) + a_{f540}\sin(2\pi 540t) \\ + a_{f660}\sin(2\pi 660t), \end{aligned} \tag{4}$$

where a_{f180}, a_{f300}, a_{f420}, a_{f420}, a_{f540}, and a_{f660} are the harmonic amplitudes at 180, 300, 420, 540, and 660 Hz, respectively. Throughout this project sampled versions of the continuous time signals are used (where the sampling is done at multiples of the Nyquist rate to avoid any aliasing). The main objective of this paper is to remove the nuisance parameters $f(t)$ and $h(t)$ from the received signal to find the mean square error estimate of $d(n)$ (the discrete time representation of $d(t)$), $\hat{d}(n)$. The following two schemes are applied to determine $\hat{d}(n)$:

1) Design the filter that outputs the MSE estimate of $d(n)$, $\hat{d}(n)$.
2) Design the filter that estimates the MSE estimates of $f(n)$ and $h(n)$ (the discrete time representation of $f(t)$ and $h(t)$), $\hat{f}(n)$ and $\hat{h}(n)$, respectively. Subsequently subtract $\hat{f}(n)$ and $\hat{h}(n)$ from the discrete time version of the received signal $r(t)$, $r(n)$ to come up with $\hat{d}(n)$.

Fig. 1 provides a block diagram representing the two algorithms applied in this report.

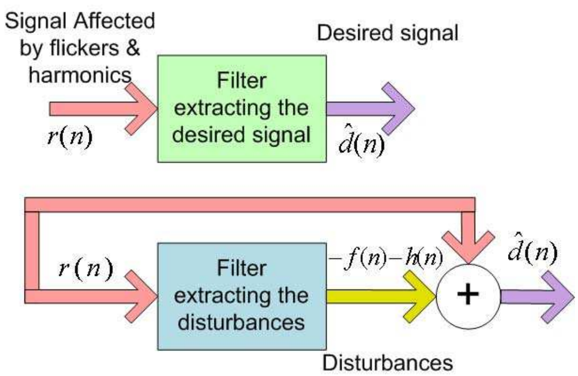

Fig. 1. The block diagram representing the two filter designs used to remove the disturbances from the received signal.

A. Filter Design Based on Estimating the Desired Signal

An FIR filter, illustrated in Fig. 2, is used to estimate $\hat{d}(n)$. Based on the filter structure the input and output relationship for the filter can be illustrated as

$$\hat{d}(n) = \sum_{k=0}^{M} w_k r(n-k), \tag{5}$$

where $\{w_0, w_1, \cdots, w_M\}$ represent the filter coefficients and M is the order of the FIR filter. Eq. (5) in vector form is represented as

$$\hat{d}(n) = \mathbf{w}^T\mathbf{r}, \tag{6}$$

where $\mathbf{w}^T$ is transpose of the $M \times 1$ vector of filter coefficients and $\mathbf{r}$ is the $M \times 1$ vector of the input parameters ($\mathbf{r} = \{r(n), r(n-1), \cdots, r(n-M)\}^T$. In general, $d(n)$ is not supposed to be known. However, in order to design the filter and to determine the optimal values of its tap weights, a short sequence of $d(n)$ must be made available. Based on the above system model the mean square error (MSE) for the above estimation problem can be defined as

$$\begin{aligned} j(n) &= E\left[(d(n) - \hat{d}(n))^2\right] \\ &= E[(d(n) - \mathbf{w}^T\mathbf{r})(d(n) - \mathbf{r}^T\mathbf{w})], \end{aligned} \tag{7}$$

where $j(n)$ is also defined as the cost function. $j(n)$ can be rewritten as

$$j(n) = E\left[(d^2(n)] - 2\mathbf{w}^T E[\mathbf{r}d(n)] + \mathbf{w}^T E[\mathbf{r}\mathbf{r}^T]\mathbf{w}\right]. \tag{8}$$

Assuming that the input and the desired sequence are stationary zero-mean random processes, Eq. (8) can be modified as follows

$$j(n) = \sigma_d^2 - 2\mathbf{w}^T\mathbf{p} + \mathbf{w}^T\mathbf{R}\mathbf{w}, \tag{9}$$

where σ_d^2 is the variance of $d(n)$, $\mathbf{p}$ is the cross correlation vector between the input sequence and the desired sequence and is expressed as

$$\mathbf{p} = E[\mathbf{r}d(n)] = \begin{pmatrix} E[r(n)d(n)] \\ E[r(n-1)d(n)] \\ E[r(n-2)d(n)] \\ \vdots \\ E[r(n-M)d(n)] \end{pmatrix}, \tag{10}$$

and matrix $\mathbf{R}$ is the autocorrelation matrix of the input sequence and is defined in Eq. (11). The objective of this design is to determine the filter coefficients, $\mathbf{w}$, such that the cost function expressed in Eq. (9) is minimized.

In Eq. (9), the cost function is a quadratic function of $\mathbf{w}$ and can be minimized by taking its gradient with respect to $\mathbf{w}$ and setting the results to zero. The gradient of $j(n)|_{\mathbf{w}}$ is represents as

$$\bigtriangledown|_{\mathbf{w}}\mathbf{j}(n) = -\mathbf{p} + 2\mathbf{R}\mathbf{w} = 0. \tag{12}$$

Taking the second gradient of $\mathbf{j}$(n) in Eq. (12) with respect to $\mathbf{w}$ results in the Hessian matrix $\mathbf{H}$,

$$\bigtriangledown^2|_{\mathbf{w}}\mathbf{j}(n) = \mathbf{H} = 2\mathbf{R}. \tag{13}$$

$$\mathbf{R} = E\left[\mathbf{r}\mathbf{r}^T\right] = \begin{pmatrix} E[r^2(n)] & E[r(n)r(n-1)] & \cdots & E[r(n)r(n-M)] \\ E[r(n-1)r(n)] & E[r^2(n-1)] & \cdots & E[r(n-1)r(n-M)] \\ \vdots & \vdots & \vdots & \vdots \\ E[r(n-M)r(n)] & E[r(n-M)r(n-1)] & \cdots & E[r^2(n-M)] \end{pmatrix} \tag{11}$$

where $h_{i,j}$ the ith row and jth column element of $\mathbf{H}$ is defined as

$$h_{i,j} = \frac{\partial^2 j(n)}{\partial w_i \partial w_j}. \tag{14}$$

Since the input sequence $r(n)$ is stationary, then the autocorrelation matrix, $\mathbf{R}$, is symmetric and positive semi-definite. This means that the Hessian matrix is positive semi-definite as well and consequently the solution of Eq. 12 leads to a minimum value for the cost function $j(n)$. Based on the results presented here the solution for the optimum filter tap weights is

$$\mathbf{w}_o = \mathbf{R}^{-1}\mathbf{p} \tag{15}$$

The results presented in Eq. (15) is known as the Wiener-Hopf Equation.

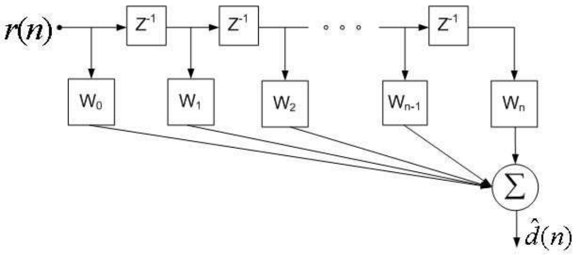

Fig. 2. The finite impulse response used to estimated $\hat{d}(n)$.

B. Filter Design Based on Estimating the Disturbances

The filter design for estimating the disturbances follows the same principles as outlined above, with the input and output relationship for the filter changed to include the harmonics and flickers instead of the desired signal. As outlined in Fig. 1 the filter design discussed above is based on estimating the desired signal $d(n)$, while in this section the objective is to estimate the disturbances, $\hat{f}(n) + \hat{f}(n)$. Since the input to the filter is still $\mathbf{r}$ the matrix $\mathbf{R}$ in Eq. (15) is the same as before. However, the cross correlation vector $\mathbf{p}$ is modified and is represented in Eq. (16).

III. Simulation Results

In this section simulation results based on the two filter designs are presented and investigated. The simulation setup is as follows: for the case of flickers a_{h50} and a_{h70} are both set to 5 and for the case of harmonics a_{f180}, a_{f300}, a_{f420}, a_{f540}, and a_{f660} are set to 30, 19, 11, 5, 3, respectively. It is important to point out that we will deviate from these values to determine the filter performance in the case of large harmonics or flickers. The sampling frequency is set to 2 kHz which is multiples of the Nyquist sampling rate and does not result in any aliasing. Fig. 3 represents the input signal $r(n)$ and the desired signal $d(n)$ for a span 1 second. It is clear that the desired signal is the 60 Hz sine wave with a constant amplitude. On the other hand the received signal suffers from amplitude fluctuations caused by the flickers and harmonics added to the signal by the transmission lines (Fig. 3).

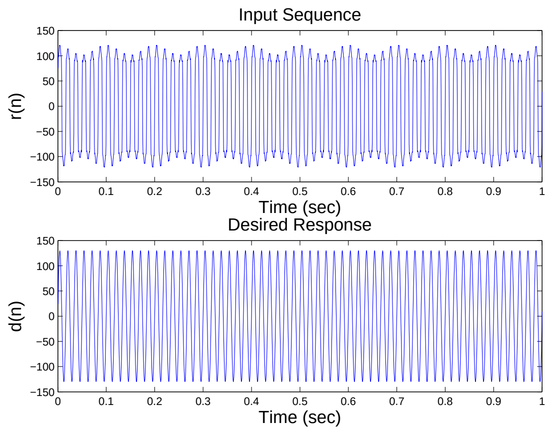

Fig. 3. The desired signal and input signal presented in a span of 1 second.

A. Simulation Result for the Filter Estimating the Desired Signal

This section presents the simulation results when the desired signal is estimated from the input signal. Using Eq. (15), the impulse response for the optimum filter has been determined and the results are plotted in Fig. 4. It is important to note that the filter order was chosen to ensure the disturbances are removed from the input signal based on the given values for the harmonics and flickers frequencies. The choice of the filter order was motivated by trial and error and is not based on analytical results. Fig. 5 represents the magnitude and phase response of the filter in the applicable frequency range. The following observations are made from this figure:

$$\mathbf{p} = E\left[\mathbf{r}\left(f(n) + h(n)\right)\right] = \begin{pmatrix} E[r(n)f(n)] + E[r(n)h(n)] \\ E[r(n-1)f(n)] + E[r(n-1)h(n)] \\ E[r(n-2)f(n)] + E[r(n-2)h(n)] \\ \vdots \\ E[r(n-M)f(n)] + E[r(n-M)h(n)] \end{pmatrix}. \quad (16)$$

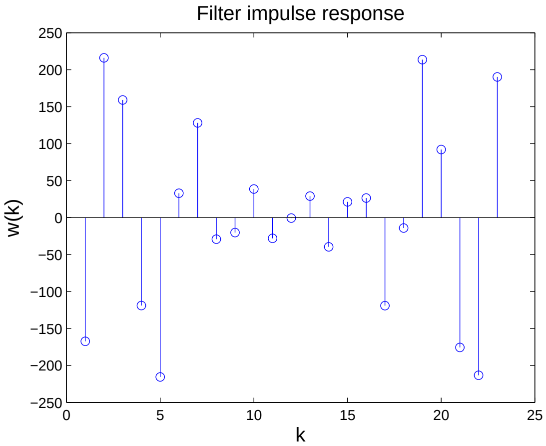

Fig. 4. The the impulse response for a filter of order 23 estimating the desired signal.

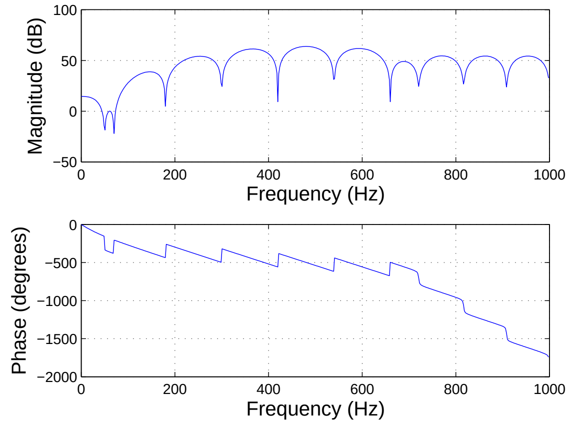

Fig. 5. Frequency and phase response of the filter outlined in Fig. 4.

1.1 The filter is a linear phase filter which is a desired characteristic (A linear phase filter has constant group delay, where all frequency components have equal delay times, resulting in no distortion due to frequency selectivity).
1.2 The magnitude response has considerable dips at the flicker and harmonic frequencies, eliminating the effect of the both distortions.
1.3 The magnitude response also demonstrates that even though the flicker frequencies are considerably attenuated, the same is not true for the harmonic frequencies. As a matter of fact, the magnitude response at 180, 300, 420, 540, and 660 Hz is considerably higher than the magnitude response and 60 Hz which is the desired signal frequency. Therefore, the designed filter is not capable of removing the effect of large harmonics and will pass them through.
1.4 The attenuations for the harmonics and flickers occur over a very narrow frequency band, making the design susceptible to significant performance degradation when the harmonics and flickers slightly deviate from expected frequencies.
1.5 The output of the filter requires some lag to stabilize as outlined in Fig. 7 and the delay required for the filter response to alleviate is increased with the filter order.

Fig. 6 represents the time response of the filter to the input $r(n)$. As expected the output of the filter is a smooth sine wave at 60 Hz and does not suffer from the effect of any of the harmonics and flickers in the original signal. As pointed out in 1.5 the output of the filter requires some lag to stabilize. This is due to the fact that filter requires a minimum of 23 samples of the input signal before it can produce an stable output and this lag is increased with the size of the filter which is a major drawback regarding large filter order sizes.

B. Simulation Result for the Filter Estimating the Disturbances

This section focuses on the design of the filter that extracts the disturbances first. As outlined in the previous section when the filter is designed to estimated the desired signal the high frequency harmonics are not attenuated much compared to the desired signal, resulting in large harmonics to negatively affect the filter output. The current design seeks to address this shortcoming by first extracting the disturbances.

Fig. 8 represents the impulse response for a filter of order 23. Eq. 15 combined with Eq. 16 are used to determine the filter coefficients and the order of the filter is kept the same to keep the comparisons between both algorithms fair. Fig. 9 illustrates the frequency and phase response of the filter over the operating bandwidth. Based on the phase and frequency response of the filter the following observations are made:

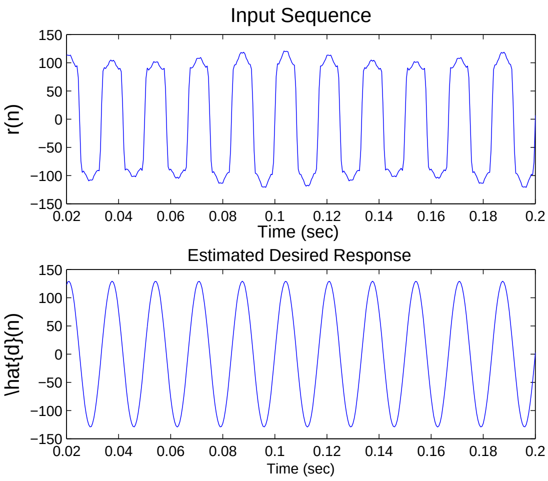

Fig. 6. The input signal to the filter and the estimated signal at the output of the filter represented in Fig. 4.

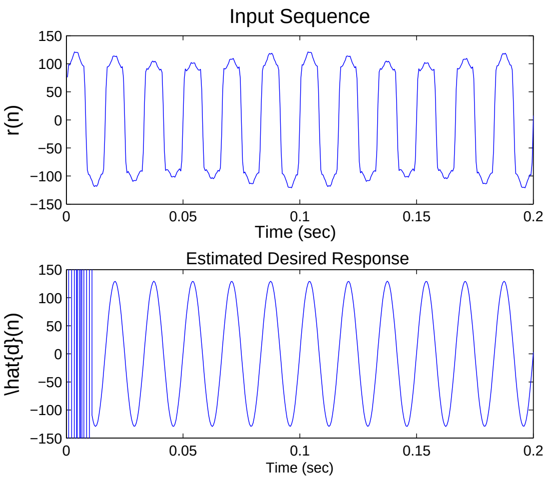

Fig. 7. The lag required for the filter response to stabilize.

2.1 Similar to the results in Fig. 5, the filter is linear phase and does not result in any distortion, which is a desired property.

2.2 The magnitude response demonstrates that the filter significantly attenuates the desired signal at 60 Hz and outputs the harmonic and flicker components of the input signal.

2.3 The difference between the magnitude response at 60Hz and the magnitude at the flicker and harmonic frequencies demonstrates that the filter does not suffer from the previous design's shortcomings and is capable of eliminating the effect of large harmonic and flickers.

Fig. 10 represents the input to filter, the output of the filter (the disturbances), and the estimated desired signal. Similar to the previous filter design the estimated desired signal has a constant amplitude at 60 Hz.

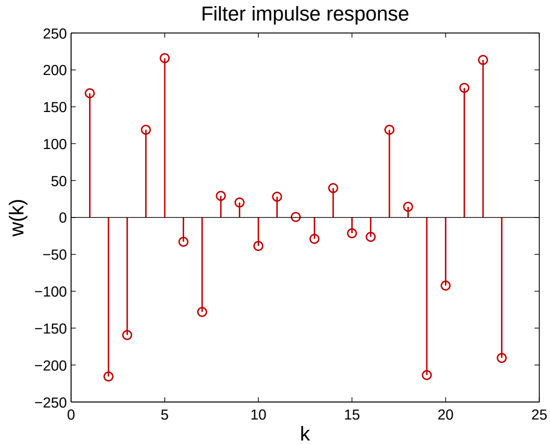

Fig. 8. The the impulse response for a filter of order 23 estimating the disturbances.

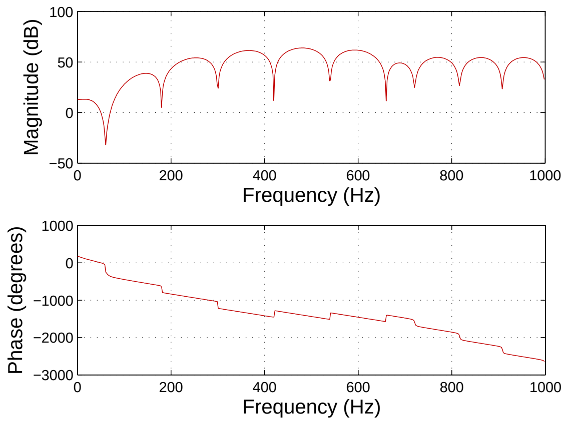

Fig. 9. Frequency and phase response of the filter outlined in Fig. 8.

C. Simulation Result for Both Filter Designs in the presence of Excessive Disturbance

It is important to examine the effect of large harmonics and flickers on the overall performance of the filter. The simulation setup for this investigation is as follows: the flicker amplitudes a_{h50} and a_{h70} are both set to 50 and the harmonic magnitudes a_{f180}, a_{f300}, a_{f420}, a_{f540}, and a_{f660} are set to 90, 130, 50, 45, 30, respectively. The simulation results for both algorithms are presented in Figs. 11 and 12. After examining the frequency responses in Figs. 5 and Figs. 9, we can conclude that the filter design based on the second algorithm performs considerably better than the first algorithm in the presence of large harmonics and flickers (as predicted in 1.3 and 2.3). Fig. 5 demonstrates that the first design is inefficient in the case of excessive

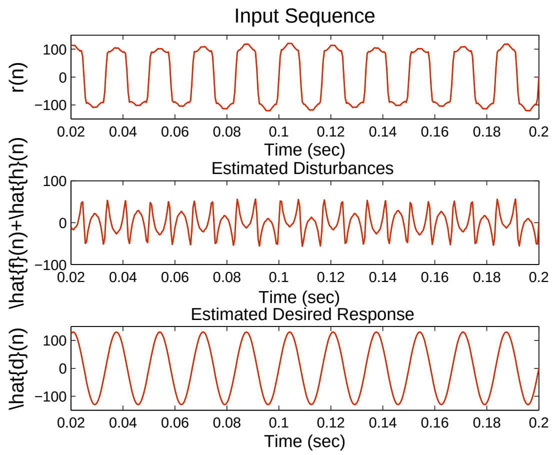

Fig. 10. The input signal to the filter and the estimated signal at the output of the filter represented in Fig. 8.

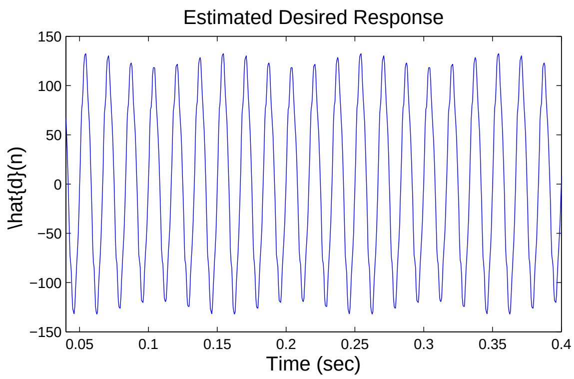

Fig. 11. The estimated signal desired signal in presence of large harmonics and flickers for the first algorithm.

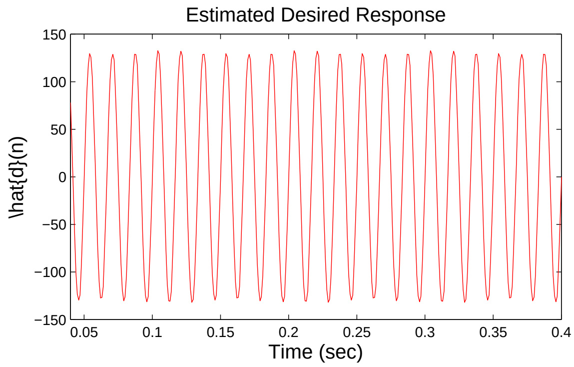

Fig. 12. The estimated signal desired signal in presence of large harmonics and flickers for the second algorithm.

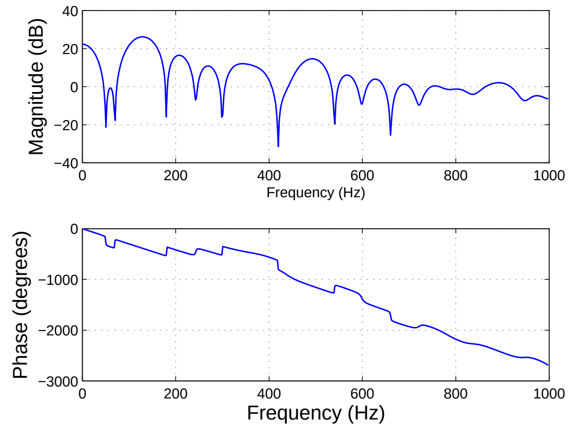

Fig. 13. The frequency and phase response of the filter design estimating the desired signal, filter order=35.

disturbances and the resulting estimated signal shows significant amplitude fluctuations.

D. Effect of Filter Order

In this section the effect of filter order on the performance of the estimator for both filter designs is investigated. It is expected that as the filter order is increased the filter can estimate the desired signal more accurately and the effect of harmonics and flickers can be mitigated more significantly. Figs. 13 and 14 represent the frequency and phase response corresponding to a filter of order 35 and 100, respectively. Comparing the results in both figures with the results presented in Fig. 5 the following observations are deduced:

3.1 As the filter order is increased the effect of harmonics and flickers are attenuated more and the filter is capable of dealing with large harmonics and flickers more appropriately.

3.2 The linear phase property of the filter is greatly affected as the filter order is increased. At filter order of 35 the filter is still linear phase, however as the filter order increases (filter order=100) the linear phase structure of the filter is not preserved, thus distorting the estimated desired signal.

3.3 Larger filter orders also increases the size of the autocorrelation matrix R in Eq. (11), making R ill-conditioned. The inverse of an ill-conditioned matrix does not exist, making Eq. (15) unsolvable.

3.4 Finally, the higher the filter order the more lags are required for the filter output to stabilize Figs. 15 and 16.

Figs. 15 and 16 represent the time domain response of the filter for filter orders of 35 and 100, respectively. As pointed out in 3.2 the filter output in Fig 16 does not resemble a sine wave and suffers from significant distortion.

The filter order investigation performed for the previous filter design is also carried out when the filter estimates the

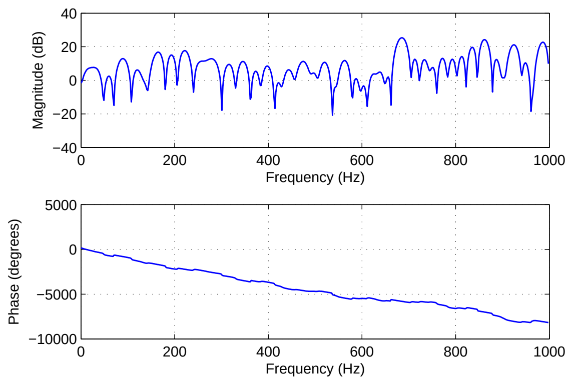

Fig. 14. The frequency and phase response of the filter design estimating the desired signal, filter order=100.

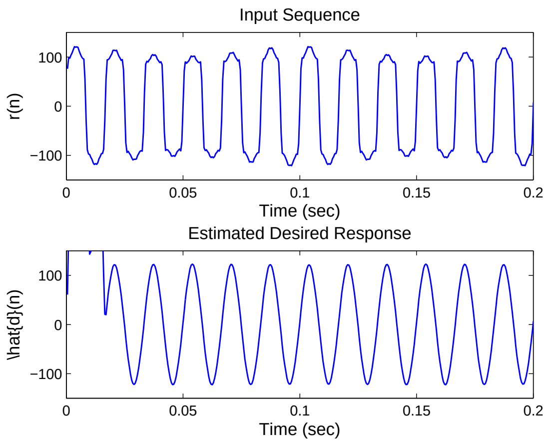

Fig. 15. The input signal to the filter and the estimated signal at the output of the filter, filter order=35.

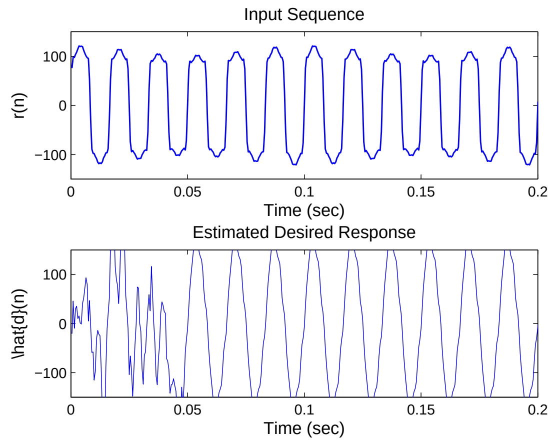

Fig. 16. The input signal to the filter and the estimated signal at the output of the filter, filter order=100.

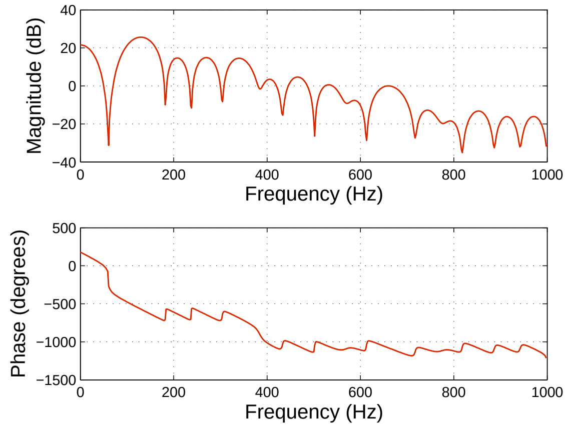

Fig. 17. The frequency and phase response of the filter design estimating the desired signal, filter order=35.

disturbances first. The frequency response of the filter for orders of 35 and 100 are illustrated in Figs. 17 and 18. Comparing the results in Figs.17 and 18 and the results in Fig. 9, the same set of conclusions as the ones outlined in 3.1-3.4 can be made. The time response of the first design for orders of 35 and 100 (Figs. 15 and 16) is significantly different from time response of the second design for the same filter orders (Figs. 19 and 20). Increasing the filter order in the first design distorts the estimated desired signal. However, in the second algorithm where the filter estimates the disturbances first this is not the case since the desired signal is not affected by the filter order at all and the objective of the filter is to estimate the flickers and harmonics and remove the desired signal. This is a significant benefit for the second algorithm since it allows for higher filter orders to be used when the disturbances are dominant and need to be mitigated significantly.

IV. Conclusion

In this paper signal estimation in the case of a power signal affected by flickers and harmonics was investigated. The Wiener-Hopf equation was applied to determine the filter coefficients based on a short training sequence and two different algorithms were used to mitigate the effect of the disturbances. In the first scheme the desired signal was first estimated from the noisy input signal and in the second algorithm the disturbances were first estimated and then removed from the noisy input signal. Both schemes are capable of removing the effect of harmonics and flickers, with the second algorithm being considerably more robust when the input signal is affected by flickers and harmonics with large amplitudes. We also examined the effect of filter order on the overall system performance and determined that a minimum filter order of 20 is required for the algorithm to operate properly. Furthermore, very large filter orders need

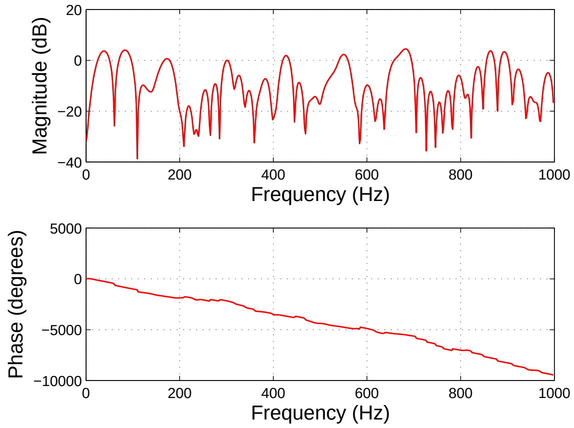

Fig. 18. The frequency and phase response of the filter design estimating the desired signal, filter order=100.

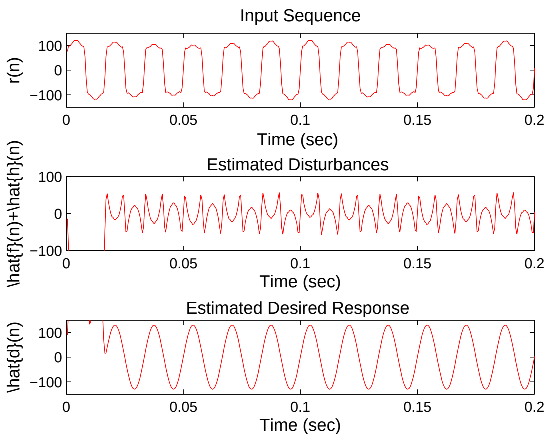

Fig. 19. The input signal to the filter and the estimated signal at the output of the filter, filter order=35.

to be avoided because it result in non-linear phase filters, which potential results in disturbances in the output of the filter, especially in the case of the first algorithm. Based on the investigation and simulations performed in this report, it can be concluded that filters designed based on Wiener-Hopf equation can be an effective way to improve the signal quality in the case power signals.

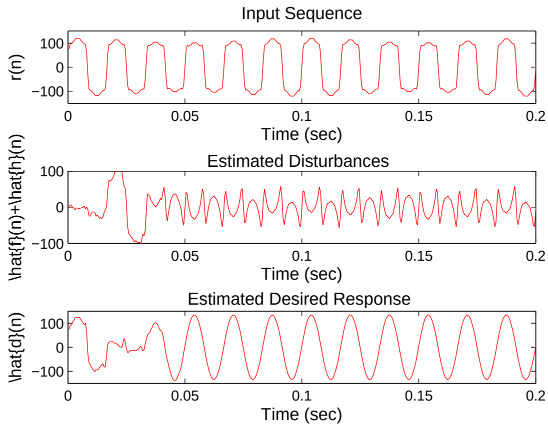

Fig. 20. The input signal to the filter and the estimated signal at the output of the filter, filter order=100.

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