Estimation, Training, and Effect of Timing Offsets in Distributed Cooperative Networks

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***Abstract*— Successful collaboration in cooperative networks requires accurate estimation of multiple timing offsets. When combined with signal processing algorithms, the estimated timing offsets can be applied to mitigate the resulting inter-symbol interference (ISI). This paper seeks to address timing synchronization in distributed multi-relay amplify-and-forward (AF) and decode-and-forward (DF) relaying networks, where timing offset estimation using a training sequence is analyzed. First, training sequence design guidelines are presented that are shown to result in improved estimation performance. Next, two iterative estimators are derived that can determine multiple timing offsets at the destination. The proposed estimators have a considerably lower computational complexity while numerical results demonstrate that they are accurate and reach or approach the Cramer-Rao lower bound (CRLB).**

I. Introduction

SYNCHRONOUS cooperative communication systems have been shown to result in multiplexing and diversity gain [1]–[3]. However, effective cooperation requires synchronization parameters such as timing offset and frequency offset to be accurately estimated. Even though frequency synchronization is addressed in [4], [5], relatively little attention has been paid to the topic of timing synchronization.

Cooperative systems are affected by multiple timing offsets, due to simultaneous transmissions from multiple nodes with different oscillators and different channel delays. The presence of timing offset, results in *inter-symbol interference (ISI)* and *signal to noise ratio (SNR)* loss [6], [7]. In [8] and [7] the effect of timing offset on probability of outage and *pair-wise error probability (PEP)* of *decode-and-forward (DF)* relaying cooperative networks is analyzed, respectively. Even though [7], [8] highlight the importance of time synchronization, no specific algorithms for estimation and synchronization of the overall network have been provided.

Cooperative strategies that result in full spatial diversity in the presence of imperfect timing synchronization are outlined in [9], [10]. However, the proposed schemes require timing offsets to be estimated for effective detection and equalization [9], [10].

In [11] the topic of timing synchronization in DF relaying networks is considered. Even though a *maximum-likelihood estimator (MLE)* is presented, the proposed estimator has extremely high computationally complexity. Note that the results in [11] are limited to the case of DF relaying and to achieve timing synchronization, the proposed MLE requires each relay's timing offset to not exceed one symbol timing duration, which is not justifiable in the case of cooperative networks consisting of multiple distributed relays with different oscillators. In [12] timing offset estimation in *amplify-and-forward (AF)* relaying cooperative networks is analyzed. However, the *Cramer-Rao lower bound (CRLB)* results in [12] are not in closed-form and the analysis is based on the assumption of perfect timing offset estimation and matched-filtering of the received signals at the relays [13]. The latter is not a justifiable assumption considering that [12] is seeking to address timing synchronization in distributed cooperative networks [13]. Moreover, unlike the results in [12], for AF relaying networks the received signals at the relays are not matched-filtered to reduce complexity and ease the deployment of relays [1]–[3]. Finally, [11], [12] do not provide any insight into the effect of training sequences on timing offset estimation performance.

This paper first quantitatively determines the effect of training sequence on timing offset estimation in multi-relay cooperative networks using the CRLBs in [14]. Next, two iterative multiple timing offset estimators are proposed that transform the R-dimensional estimation problem into R single parameter estimation problems that are then solved using the 1-dimensional MLE and *Gardner's detector (GD)*. As a result, the proposed estimators significantly reduce the computational complexity associated with timing synchronization in cooperative networks. Numerical results illustrate that the proposed estimators are accurate over a wide range of SNR values.

This paper is organized as follows: in Section II, the training signal model for cooperative networks in the presence of timing offsets is formulated. In Section III training sequence design guidelines are outlined. Section IV derives the iterative multiple timing offset estimators and investigates their complexity. Section V presents numerical and simulation results.

Notation: italic letters (x) are scalars, bold letters ($\mathbf{x}$) are vectors, bold upper case letters ($\mathbf{X}$) are matrices, $[\mathbf{X}]_{k,m}$ represents the kth row and mth column element of $\mathbf{X}$, $\odot$ stands for Schur (element-wise) product, and $(\cdot)^*$, $(\cdot)^T$, $(\cdot)^H$, and $\text{Tr}(\cdot)$ denote conjugate, transpose, conjugate transpose, and trace, respectively.

II. System Model

A half-duplex cooperative network consisting of a source and destination pair and a cluster of R relays is considered (see Fig. 1). Multiple timing offset estimation using a training sequence is analyzed, where during the *training interval* the timing offsets corresponding to the R relay nodes are estimated [14]. These estimates can be applied in the *data transmission interval* to eliminate *inter-symbol interference (ISI)* (see Fig. 1). Throughout this paper the following set of assumptions are considered:

1) In *Phase I*, the source broadcasts its *training sequence (TS)* to the relays. In *Phase II*, the relays transmit R distinct TSs *simultaneously* to the destination (see Fig. 1).

2) Timing offsets are modeled as *unknown* non-random parameters.

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3) Similar to most timing offset methods, it is assumed that nodes within the network are synchronized in frequency [11], [14], [15].
4) Quasi-static channels are considered, where the channel gains do *not* change over a frame but *change* from frame-to-frame. Each frame consists of many symbols, e.g. 1024 symbols, that are transmitted over a single time slot.

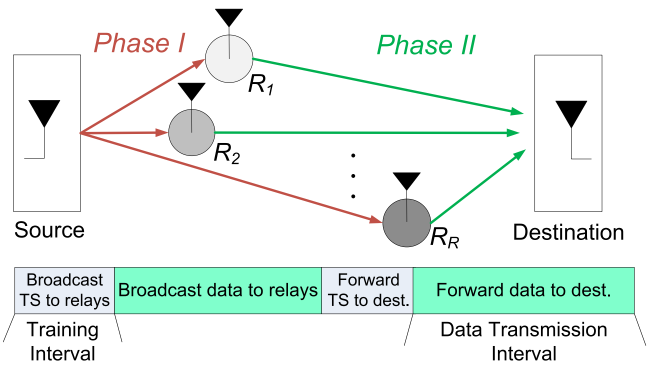

Fig. 1. The system model and scheduling diagram for training and data transmission intervals for the cooperative network.

Assumptions 2-4 are in line with previous timing offset estimation analyses in [11], [12] and are also intuitively justifiable, since the main sources of timing offset are oscillator mismatch and channel delay [6]. In addition, oscillator properties and channel delays are assumed not to change during the short TS.

A. Training Signal Model at the Relays

The ith sample of the baseband received training signal, for $i = 1, \cdots, NL$, prior to matched filtering, $r_k(i)$, at the kth relay for $k = 1, \cdots, R$ is given by

$$r_k(i) = h_k \sum_{n=0}^{L-1} t^{[s]}(n) g(iT_s - nT - \tau_k^{[sr]}T) + v_k(i), \quad (1)$$

where:

- L and T denote the length of the TS and the symbol duration, respectively, and $T = NT_s$, where T_s is the sampling time and N is the number of samples per symbol,
- $t^{[s]}(n)$ is the *known* nth training symbol broadcast from the source to the relays, $\tau_k^{[sr]}$ is the normalized timing offset from source to the kth relay, and $g(t)$ is the pulse shaping filter,
- h_k denotes the *unknown* channel gain from source to the kth relay, and
- $v_k(n)$ is *additive white Gaussian noise (AWGN)* at the kth relay with mean zero and variance $\sigma_{v_k}^2$, $\mathcal{CN}(0, \sigma_{v_k}^2)$.

Eq. (1) can be represented in matrix and vector form as

$$\mathbf{r_k} = h_k \mathbf{G_k^{[sr]}} \mathbf{t^{[s]}} + \mathbf{v_k}, \quad (2)$$

where:

- $\mathbf{r_k} \triangleq [r_k(0), \cdots, r_k(NL-1)]^T$,
- $\mathbf{t^{[s]}} \triangleq [t^{[s]}(0), \cdots, t^{[s]}(L-1)]^T$,
- $\mathbf{v_k} \triangleq [v_k(0), v_k(1), \cdots, v_k(NL-1)]^T$, and
- $\left[\mathbf{G_k^{[sr]}}\right]_{m,l} \triangleq g(mT_s - lT - \tau_k^{[sr]}T)$ is an $NL \times L$ matrix.

B. Training Signal Model for DF Relaying Networks

The DF protocol requires that the signals at the relays be decoded and timing offsets, $\boldsymbol{\tau}^{\mathbf{[sr]}} \triangleq \left[\tau_1^{[sr]}, \cdots, \tau_R^{[sr]}\right]^T$ be estimated and equalized at the relays. Therefore, $\mathbf{t^{[s]}}$ received in *phase I* is used for timing offset estimation and compensation similar to that of a point-to-point *single-input-single-output (SISO)* system [6].

Unlike *Phase I*, in *Phase II*, the superposition of the received training signals must be used to jointly estimate the timing offsets from the relays to destination, $\boldsymbol{\tau}^{\mathbf{[rd]}} \triangleq \left[\tau_1^{[rd]}, \cdots, \tau_R^{[rd]}\right]^T$. The sampled baseband received training signal model, $\mathbf{y} \triangleq [y(0), y(1), \cdots, y(NL-1)]^T$, for a DF relaying network consisting of R relays is given by

$$\mathbf{y} = \sum_{k=1}^{R} \left(f_k \mathbf{G_k^{[rd]}} \mathbf{t_k^{[r]}} \right) + \mathbf{w}, \quad (3)$$

where:

- $\tau_k^{[rd]}$ is the normalized timing offset from the kth relay to the destination and $\mathbf{G_k^{[rd]}}$ is an $NL \times L$ matrix, where $\left[\mathbf{G_k^{[rd]}}\right]_{m,l} \triangleq g(mT_s - lT - \tau_k^{[rd]}T)$,
- f_k denotes the *unknown* channel gain from the kth relay to destination,
- $\mathbf{t_k^{[r]}} \triangleq [t_k^{[r]}(0), \cdots, t_k^{[r]}(L-1)]^T$ is the kth relay's *known* transmitted TS, and
- $\mathbf{w} \triangleq [w(0), w(1), \cdots, w(NL-1)]^T$ is the zero-mean AWGN at the destination modeled as $\mathcal{CN}(0, \sigma_w^2)$.

C. Training Signal Model for AF Relaying Networks

In most practical AF cooperative networks a digital signal processing or beamforming algorithm is applied at the relays to improve the overall system's performance, e.g., [2], [3]. Therefore, to enable synchronous transmission and successful cooperation for AF networks, the relays need to estimate timing offsets from source to relays, $\boldsymbol{\tau}^{\mathbf{[sr]}}$. In addition, to estimate the timing offset corresponding to each relay, the TS transmitted from the kth relay needs to be distinct. Hence, the baseband processing structure in Fig. 2 at the relays [14] is proposed. The design of the timing corrector block in Fig. 2 is outlined in [6].

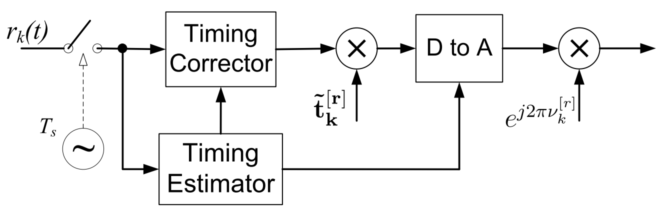

Fig. 2. Block diagram of the proposed baseband processing at the kth relay for AF relaying networks during the training interval.

Note that even though the proposed processing structure in Fig. 2 requires additional hardware at the relays, the overall relay structure, in the case of AF relaying, is still considerably simpler than that of DF relaying networks and the model in [12].

The sampled baseband representation of the received training signal model at the destination in *Phase II* is given by

$$y(i) = \sum_{k=1}^{R}\sum_{n=0}^{L-1} \zeta_k f_k \tilde{t}_k^{[r]}(i) \underbrace{r_k(i)}_{\textbf{Eq. (1)}} + w(i) \tag{4a}$$

$$= \underbrace{\sum_{k=1}^{R}\sum_{n=0}^{L-1} \zeta_k f_k h_k t^{[s]}(n) \tilde{t}_k^{[r]}(i) g(iT_s - nT - \tau_k^{[rd]}T)}_{\textbf{desired signal}}$$

$$+ \underbrace{\sum_{k=1}^{R} \zeta_k f_k \tilde{v}_k(i) + w(i)}_{\textbf{overall noise}}, \quad \text{for } 0 \le i \le NL-1 \tag{4b}$$

where:

- $\tilde{v}_k(i) \triangleq v_k(i)\tilde{t}_k^{[r]}(i)$ and $\tilde{t}_k^{[r]}(i)$ is the ith symbol of the kth relay's TS in the case of AF relaying, and
- ζ_k is a scaling factor that satisfies the kth relay's power constraint.

Eq. (4b) follows from (4a) since the received signal vector at the kth relay, $\mathbf{r_k}$, is amplified and forwarded without being decoded and due to the application of the relay processing indicated in Fig. 2. Eq. (4b) can be rewritten in matrix and vector form as

$$\mathbf{y} = \sum_{k=1}^{R} \zeta_k f_k h_k \left(\mathbf{G_k^{[rd]}} \mathbf{t^{[s]}}\right) \odot \tilde{\mathbf{t}}_\mathbf{k}^{[\mathbf{r}]} + \sum_{k=1}^{R} \zeta_k f_k \tilde{\mathbf{v}}_\mathbf{k} + \mathbf{w}, \tag{5}$$

where $\tilde{\mathbf{t}}_\mathbf{k}^{[\mathbf{r}]} \triangleq \left[\tilde{t}_k^{[r]}(0),, \cdots, \tilde{t}_k^{[r]}(NL-1)\right]^T$ is defined in (4b) and $\tilde{\mathbf{v}}_\mathbf{k} \triangleq [\tilde{v}_k(0), \tilde{v}_k(1), \cdots, \tilde{v}_k(NL-1)]^T$.

Note that the main sources of timing offsets, $\boldsymbol{\tau}^{[\mathbf{rd}]}$ for both DF and AF relaying are timing offset estimation error at the relays, oscillator mismatch, and channel delay.

III. Training Sequence Design

The CRLBs derived in [14] are applied in this section to quantitatively determine the effect of *training sequence (TS)* on timing offset estimation performance and to propose new training sequence design guidelines. For notational clarity, $(\cdot)^{[\mathbf{DF}]}$ and $(\cdot)^{[\mathbf{AF}]}$ are used instead of $(\cdot)^{[\mathbf{rd}]}$, for DF and AF relaying, respectively.

Given that the CRLB for the joint estimation of timing offsets and channel delays for multi-relay cooperative networks is too complex and provides little insight on the effect of training sequence and network topology on timing offset estimation, the CRLB expressions are derived based on the assumption of *known* channel gains. The CRLB for the joint estimation of timing offsets at the destination for DF relaying networks is given by [14]

$$\mathbf{CRLB_{DF}} = \underbrace{\frac{\sigma_w^2}{2} \text{Re}\left\{\mathbf{D_f}^{-1}\left(\left(\boldsymbol{\Delta}^{[\mathbf{DF}]}\right)^H \boldsymbol{\Delta}^{[\mathbf{DF}]}\right)^{-1}\left(\mathbf{D_f}^H\right)^{-1}\right\}}_{\triangleq \mathbf{FIM}_{\mathbf{DF}}^{-1}}, \tag{6}$$

where $\mathbf{D_f} \triangleq \text{diag}(f_1, f_2, \cdots, f_R)$ is an $R \times R$ matrix, $\boldsymbol{\Delta}^{[\mathbf{DF}]} \triangleq \left[\boldsymbol{\delta}_\mathbf{1}^{[\mathbf{DF}]}, \boldsymbol{\delta}_\mathbf{2}^{[\mathbf{DF}]}, \cdots, \boldsymbol{\delta}_\mathbf{R}^{[\mathbf{DF}]}\right]$ is an $NL \times R$ matrix, $\boldsymbol{\delta}_\mathbf{k}^{[\mathbf{DF}]} \triangleq \partial \boldsymbol{\xi}_\mathbf{k}^{[\mathbf{DF}]}/\partial \tau_k^{[\text{DF}]} = \partial \mathbf{G}_\mathbf{k}^{[\mathbf{DF}]}/\partial \tau_k^{[\text{DF}]} \mathbf{t}_\mathbf{k}^{[\mathbf{r}]}$, and $\boldsymbol{\xi}_\mathbf{k}^{[\mathbf{DF}]} \triangleq \mathbf{G}_\mathbf{k}^{[\mathbf{DF}]} \mathbf{t}_\mathbf{k}^{[\mathbf{r}]}$. In the case of AF relaying networks the CRLB is given by

$$\mathbf{CRLB_{AF}} = \frac{\sigma_n^2}{2} \text{Re}\left\{\mathbf{D}_{\boldsymbol{\alpha}}^{-1}\left(\left(\boldsymbol{\Delta}^{[\mathbf{AF}]}\right)^H \boldsymbol{\Delta}^{[\mathbf{AF}]}\right)^{-1}\left(\mathbf{D}_{\boldsymbol{\alpha}}^H\right)^{-1}\right\}, \tag{7}$$

where $\sigma_n^2 \triangleq \sum_{k=1}^{R}\left(|\beta_k|^2 \sigma_{\tilde{v}_k}^2\right) + \sigma_w^2$, $\mathbf{D}_{\boldsymbol{\alpha}} \triangleq \text{diag}(\alpha_1, \cdots, \alpha_R)$ is an $R \times R$ matrix, $\alpha_k \triangleq \zeta_k f_k h_k$, $\beta_k \triangleq \zeta_k f_k$, $\boldsymbol{\Delta}^{[\mathbf{AF}]} \triangleq \left[\boldsymbol{\delta}_\mathbf{1}^{[\mathbf{AF}]}, \boldsymbol{\delta}_\mathbf{2}^{[\mathbf{AF}]}, \cdots, \boldsymbol{\delta}_\mathbf{R}^{[\mathbf{AF}]}\right]$ is an $NL \times R$ matrix, $\boldsymbol{\delta}_\mathbf{k}^{[\mathbf{AF}]} \triangleq \partial \boldsymbol{\xi}_\mathbf{k}^{[\mathbf{AF}]}/\partial \tau_k^{[\text{AF}]} = \left(\frac{\partial \mathbf{G}_\mathbf{k}^{[\mathbf{AF}]}}{\partial \tau_k^{[\text{AF}]}} \mathbf{t}^{[\mathbf{s}]}\right) \odot \tilde{\mathbf{t}}_\mathbf{k}^{[\mathbf{r}]}$, and $\boldsymbol{\xi}_\mathbf{k}^{[\mathbf{AF}]} \triangleq \left(\mathbf{G}_\mathbf{k}^{[\mathbf{AF}]} \mathbf{t}^{[\mathbf{s}]}\right) \odot \tilde{\mathbf{t}}_\mathbf{k}^{[\mathbf{r}]}$.

Let us consider the case of DF relaying first.

Theorem 1: The $\mathbf{CRLB_{DF}}$ in (6) is minimized, when the matrix $\boldsymbol{\Omega} \triangleq \left(\boldsymbol{\Delta}^{[\mathbf{DF}]}\right)^H \boldsymbol{\Delta}^{[\mathbf{DF}]}$ is diagonal.

Proof: According to (6) minimizing the CRLB for the estimation of $\boldsymbol{\tau}^{[\mathbf{DF}]}$ is equivalent to minimizing the trace of the matrix $\mathbf{CRLB_{DF}}$. Moreover, based on the results in [16], for an $M \times M$ positive definite matrix $\mathbf{X}$ the following holds,

$$\text{Tr}\left[\mathbf{X^{-1}}\right] \ge \sum_{j=1}^{M} \frac{1}{[\mathbf{X}]_{jj}}, \tag{8}$$

with equality if $\mathbf{X}$ is diagonal. Let us assume that the optimum $\boldsymbol{\Omega}$ that minimizes $\text{Tr}\left[\mathbf{CRLB_{DF}}\right]$ is not diagonal. Then, we can conclude that $\mathbf{FIM_{DF}}$ in (6) is also not diagonal. Using (8) we obtain that

$$\text{Tr}\left[\mathbf{CRLB_{DF}}(\boldsymbol{\tau}^{[\mathbf{DF}]})\right] = \text{Tr}\left[\left(\mathbf{FIM_{DF}}\right)^{-1}\right] \ge \sum_{j=1}^{M} \frac{1}{\left[\mathbf{FIM_{DF}}\right]_{jj}}. \tag{9}$$

According to (9), there exists a matrix $\overline{\mathbf{FIM}}_\mathbf{DF} = \text{diag}\left\{\mathbf{FIM_{DF}}\right\}$ that results in a lower $\text{Tr}\left[\mathbf{CRLB_{DF}}\right]$. This leads to a contradiction. Hence, the optimum $\boldsymbol{\Omega}$ must be diagonal. Using (7) and similar steps as above it can be shown that $\mathbf{CRLB_{AF}}$ is also minimized when $\left(\boldsymbol{\Delta}^{[\mathbf{AF}]}\right)^H \boldsymbol{\Delta}^{[\mathbf{AF}]}$ is diagonal.

According to the CRLB expressions in (6) and (7), if the training sequences transmitted from the relays are linearly dependent and the timing offsets $\tau_1^{[rd]} = \tau_2^{[rd]} = \cdots = \tau_R^{[rd]}$, the matrices $\left(\boldsymbol{\Delta}^{[\mathbf{DF}]}\right)^H \boldsymbol{\Delta}^{[\mathbf{DF}]}$ and $\left(\boldsymbol{\Delta}^{[\mathbf{AF}]}\right)^H \boldsymbol{\Delta}^{[\mathbf{AF}]}$ become singular and the CRLBs approach infinity. This indicates that there does not exist an unbiased estimator that can determine the timing offsets at the destination. However, note that application of orthogonal training sequences ensures that matrices $\left(\boldsymbol{\Delta}^{[\mathbf{DF}]}\right)^H \boldsymbol{\Delta}^{[\mathbf{DF}]}$ and $\left(\boldsymbol{\Delta}^{[\mathbf{AF}]}\right)^H \boldsymbol{\Delta}^{[\mathbf{AF}]}$ are diagonal when the timing offset values are the same, which according to Theorem 1, lowers the CRLB. This indicates that timing offsets, $\boldsymbol{\tau}^{[\mathbf{rd}]}$, can be accurately estimated at the destination using orthogonal training sequences even if they are the same or close to one another.

In addition to the results above, numerical analyses observed in Section V indicate that under the assumption of Nyquist transmitted pulses, $g(t)$, training sequences that alternate in sign from one symbol to another such that

$$t_k^{[r]}(n) = (-1)^n \quad \text{for} \quad 1 \le k \le R \tag{10}$$

result in lower CRLB.

IV. Proposed Timing Offset Estimator

In this section a brief overview of the MLE for multiple timing offset estimation [11], [18] is first provided and then *iterative-MLE (I-MLE)* and *iterative-Gardner detector (I-GD)* are derived. For readability purposes timing offset estimation in DF relaying networks is discussed first and $\boldsymbol{\tau}$ is used instead of $\boldsymbol{\tau}^{[\mathbf{DF}]}$ below.

A. MLE for Multiple Timing Offset Estimation

Eq. (3) can be rewritten in matrix and vector form as

$$\mathbf{y} = \mathbf{\Xi}^{[\mathbf{DF}]}(\boldsymbol{\tau})\mathbf{f} + \mathbf{w}, \quad (11)$$

where $\mathbf{\Xi}^{[\mathbf{DF}]}(\boldsymbol{\tau}) \triangleq \left[\boldsymbol{\xi}_{\mathbf{1}}^{[\mathbf{DF}]}, \boldsymbol{\xi}_{\mathbf{2}}^{[\mathbf{DF}]}, \cdots, \boldsymbol{\xi}_{\mathbf{R}}^{[\mathbf{DF}]}\right]$ and $\boldsymbol{\xi}_{\mathbf{k}}^{[\mathbf{DF}]}$, and $\boldsymbol{\xi}_{\mathbf{k}}^{[\mathbf{DF}]}$ is defined in (6). Since the vector of received training signals, $\mathbf{y}$, is a Gaussian random variable, the joint *log-likelihood function (LLF)*, $\gamma(\boldsymbol{\tau}, \mathbf{f})$, is proportional to

$$\gamma(\boldsymbol{\tau}, \mathbf{f}) = \left\|\mathbf{y} - \mathbf{\Xi}^{[\mathbf{DF}]}\boldsymbol{\eta}\right\|^2. \quad (12)$$

It is a well known that for a given $\boldsymbol{\tau}$, the minimizer of (12) is

$$\hat{\mathbf{f}} = \left(\left(\mathbf{\Xi}^{[\mathbf{DF}]}\right)^H \mathbf{\Xi}^{[\mathbf{DF}]}\right)^{-1} \left(\mathbf{\Xi}^{[\mathbf{DF}]}\right)^H \mathbf{y}. \quad (13)$$

Inserting (13) into (12), $\hat{\boldsymbol{\tau}}$ can be obtained as

$$\hat{\boldsymbol{\tau}} = \arg\max_{\boldsymbol{\tau}} \mathbf{y}^H \mathbf{\Xi}^{[\mathbf{DF}]} \left(\left(\mathbf{\Xi}^{[\mathbf{DF}]}\right)^H \mathbf{\Xi}^{[\mathbf{DF}]}\right)^{-1} \left(\mathbf{\Xi}^{[\mathbf{DF}]}\right)^H \mathbf{y}, \quad (14)$$

where the set of possible timing offsets, $\boldsymbol{\tau} = \{\tau_1, \cdots, \tau_R\}$ can be represented as

$$\tau_k \in \{-\epsilon_k : \Delta\varsigma_k : \epsilon_k\} \quad \text{for } 1 \le k \le R, \quad (15)$$

with $[-\epsilon_k, \epsilon_k)$ and $\Delta\varsigma_k$ denoting the *estimation range* and *step size* for the kth relay's timing offset, respectively. Based on (14) the following remarks are in order:

Remark 1: The maximization in (14) is very complex, given that it requires carrying out large matrix multiplications and inversion. Moreover, to accurately estimate each relay's timing offset the step size in (14) needs to be small, $\Delta\varsigma_k \le 10^{-4}$ for $1 \le k \le R$, increasing the complexity of the exhaustive search.

Remark 2: The computationally complex MLE needs to be carried out every time the nodes within the network are re-synchronized.

B. I-MLE for DF Networks

Note that the training signal model in (3) can be rewritten as

$$\mathbf{y} = \underbrace{f_m \mathbf{G}_{\mathbf{m}}^{[\mathbf{DF}]}\mathbf{t}_{\mathbf{m}}^{[\mathbf{r}]}}_{\textbf{desired term}} + \underbrace{\sum_{k=1, k\neq m}^{R} \left(f_k \mathbf{G}_{\mathbf{k}}^{[\mathbf{DF}]}\mathbf{t}_{\mathbf{k}}^{[\mathbf{r}]}\right)}_{\textbf{interference}} + \underbrace{\mathbf{w}}_{\textbf{noise}}. \quad (16)$$

Eq. (16) shows that while estimating the mth relay's timing offset the training signals from the remaining relays act as an interference. By eliminating the interference term in (16), the mth relay's timing offset can be estimated similar to that of a point-to-point system. Table I summarizes the proposed I-MLE algorithm, where $(\cdot)^{[\mathbf{o}]}$ represents the oth iteration.

C. I-GD for DF Networks

Gardner's detector (GD) [15] is an effective timing offset estimator that has been widely applied due to its simplicity. However, the application of GD in the case of cooperative networks is complicated due to the presence of multiple timing offsets. To address this shortcoming we propose the iterative Gardner detector (I-GD).

Using the mth relay's training signal, $\mathbf{q_m}$ in (17) the output of the GD, $\varpi_m(n)$ is given by

$$\varpi_m(n) = \text{Re}\left\{q_m^*(T_{n-\frac{1}{2}})\left[q_m(T_n) - q_m(T_{n-1})\right]\right\}, \quad (19)$$

TABLE I

I-MLE TIMING OFFSET ESTIMATOR

Step 1)	**Initialization** • Set the timing offsets to zero, $(\hat{\boldsymbol{\tau}})^{[\mathbf{0}]} = \mathbf{0}$. • Use the alternating projection method [18] and a large step size, e.g.,$\Delta\iota_k = 5 \times 10^{-2}$ for $1 \le k \le R$, to solve (14) and calculate rough initial estimates of the timing offsets, $(\hat{\boldsymbol{\tau}})^{[\mathbf{1}]}$. • Calculate the initial channel gains, $\hat{\mathbf{f}}^{[\mathbf{1}]}$, using (13).

Step 2)	**Iteration**
	$o = 1$
	While $\left\|\gamma\left(\hat{\boldsymbol{\tau}}^{[\mathbf{o+1}]}, \hat{\mathbf{f}}^{[\mathbf{o+1}]}\right) - \gamma\left(\hat{\boldsymbol{\tau}}^{[\mathbf{o}]}, \hat{\mathbf{f}}^{[\mathbf{o}]}\right)\right\| \ge \chi$ *do*
	• For $m = 1, 2, \cdots, R$
	– Compute the mth relay's training signal via
	$\mathbf{q_m} = \mathbf{y} - \sum_{k=1,k\neq m}^{R} \left(\sqrt{p_k^{[r]}}\left(\hat{f}_k\right)^{[1]}\left(\hat{\mathbf{G}}_{\mathbf{k}}^{[\mathbf{DF}]}\right)^{[\mathbf{1}]}\mathbf{t}_{\mathbf{k}}^{[\mathbf{r}]}\right)$, (17)
	where, $\left(\hat{\mathbf{G}}_{\mathbf{k}}^{[\mathbf{DF}]}\right)^{[\mathbf{1}]}$ is a function of $(\hat{\boldsymbol{\tau}})^{[\mathbf{1}]}$ and is defined in (3).
	– Using $\mathbf{q_m}$ determine the mth relay's timing offset using
	$\left(\hat{\tau}_m^{[\mathrm{DF}]}\right)^{[o+1]} = \arg\max_{\tau_m} \frac{\mathbf{q_m}^H \boldsymbol{\xi}_{\mathbf{m}}^{[\mathbf{DF}]}(\boldsymbol{\xi}_{\mathbf{m}}^{[\mathbf{DF}]})^H \mathbf{q_m}}{\left(\boldsymbol{\xi}_{\mathbf{m}}^{[\mathbf{DF}]}\right)^H \boldsymbol{\xi}_{\mathbf{m}}^{[\mathbf{DF}]}}$, (18)
	where $\Delta\varrho_m$ is the smaller step size of the 1-dimensional MLE, e.g., $\Delta\varrho_m = 10^{-4}$ for $1 \le m \le R$, $[-\varepsilon, \varepsilon)$ represents the new smaller estimation range, and $\boldsymbol{\xi}_{\mathbf{m}}^{[\mathbf{DF}]}$ is defined in (6).
	– Compute the channel gains, $(\hat{\boldsymbol{\eta}})^{[\mathbf{o+1}]}$, using (13).
	• $o = o + 1$
	end While

where T_n and $T_{n-1/2}$ represent the interpolation instances of the nth symbol and are calculated as

$$T_n = nT + \hat{\tau}_m(n), \text{ and} \quad (20)$$

$$T_{n-\frac{1}{2}} = nT - \frac{T}{2} + \frac{\hat{\tau}_m(n) + \hat{\tau}_m(n-1)}{2}, \quad (21)$$

respectively. $\hat{\tau}_m(n)$ represents the nth estimate of the mth relay's timing offset. Note that the design of the interpolator, timing controller, and loop filter for the GD are outlined in [6].

D. I-MLE and I-GD for AF Networks

Using the definitions in (7) and by combining the noise terms, (5) can be rewritten as

$$\mathbf{y} = \sum_{k=1}^{R} \alpha_k \boldsymbol{\xi}_{\mathbf{k}}^{[\mathbf{AF}]} + \mathbf{z_c}, \quad (22)$$

where the overall noise, $\mathbf{z_c} \triangleq \sum_{k=1}^{R} \beta_k \tilde{\mathbf{v}}_{\mathbf{k}} + \mathbf{w}$. Under the assumption of AWGN and mutually independent noise at the relays and destination and quasi-static frequency-flat fading channels, the covariance matrix of $\mathbf{z_c}$ can be readily determined as $\mathbf{\Sigma_{z_c}} = \left(\sum_{k=1}^{R}\left(|\beta_k|^2\sigma_{\tilde{v}_k}^2\right) + \sigma_w^2\right)\mathbf{I}$. Since for AF relaying networks the overall noise is white Gaussian and the signal model in (22) is similar to that of DF networks in (3), I-MLE and I-GD can be applied to estimate $\boldsymbol{\tau}^{[\mathbf{AF}]}$ and $\boldsymbol{\alpha}$.

V. NUMERICAL RESULTS AND DISCUSSIONS

Throughout this section the propagation loss is modeled as [6] $\beta = (d/d_0)^{-m}$ where d is the distance between the transmitter and receiver, d_0 is the reference distance, and m is the path loss exponent [6]. The following results are based on $d_0 = 1$km and $m = 2.7$, which corresponds to urban area cellular networks. The transmit pulse shaping filter, $g(t)$ is *root-raised-cosine (RRC)*. The roll-off factor is set to .22. Finally, $\sigma_{v_1}^2 = \cdots = \sigma_{v_k}^2 = \sigma_w^2$.

Numerical analysis in Fig. 3 illustrates that TSs that alternate more in sign perform considerably better. Note that TS-3, TS-5, and TS-7 alternate in sign, 3, 5, and 7 times, respectively, where as shown in Fig. 3, TS-7 with the largest number of sign alternations has the lowest CRLB.

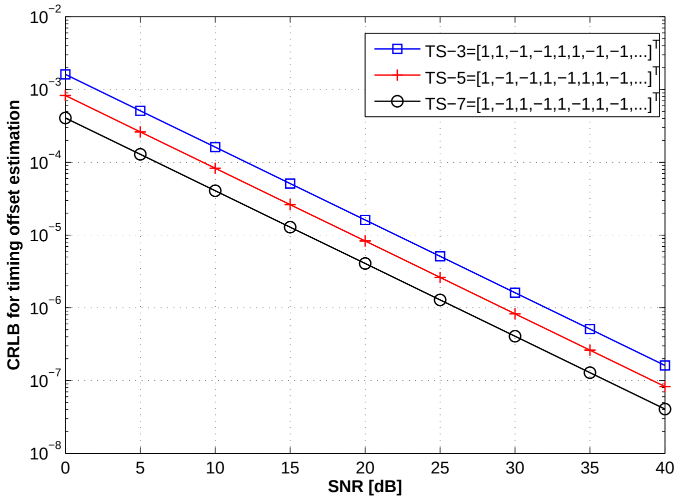

Fig. 3. Comparison of the CRLB (6) for different orthogonal TSs, demonstrating that orthogonality is not the only condition that affects timing offset estimation in cooperative networks ($R = 2$, $L = 64$, and $N = 2$).

Fig. 4 A. compares the *mean-square error (MSE)* of I-MLE and I-GD for the estimation of timing offsets in DF and AF relaying networks against the CRLB in (6) and (7), respectively. The estimation performance for the initialization step of both algorithms is also presented. The MSE for the MLE-AP algorithm [18] is not shown since it is similar to that of I-MLE. The channel gains, $\mathbf{h}$ are drawn from *independent and identically distributed (i.i.d)* zero-mean complex Gaussian processes with unit variance. For our particular channels $\mathbf{h} = [.7820+.6233i, .9474-.3203i]^T$ and $\mathbf{f} = [.2790 - .9603i, .8837 + .4681i]^T$. Without loss of generality, only the MSE for the first node timing offset, τ_1, is presented.

The results in Fig. 4 A. illustrate that I-MLE reaches the CRLB over a wide range of SNR values. On the other hand, I-GD demonstrates good performance but similar to the GD suffers from an error floor at high SNR values due to the self-noise as also shown in [6]. The threshold for the stopping criteria is set to $\chi = .001$. Fig. 4 B. shows that $\chi = .001$, results in the convergence of both I-MLE and I-GD to the true timing offsets in $2 - 5$ iterations.

VI. Conclusion

In this paper training sequence design guidelines that improve multiple timing offset estimation in cooperative networks are investigated. It is revealed that training sequences that are orthogonal to one another and alternate the most in sign from one symbol to another lower the CRLB and improve estimation performance. Two multiple timing offset estimators denoted by I-MLE and I-GD are proposed that significantly reduce the complexity and overhead associated with timing synchronization in distributed cooperative networks. Simulation results show that I-MLE reaches the CRLB over a wide range of SNR values while I-GD shows good performance.

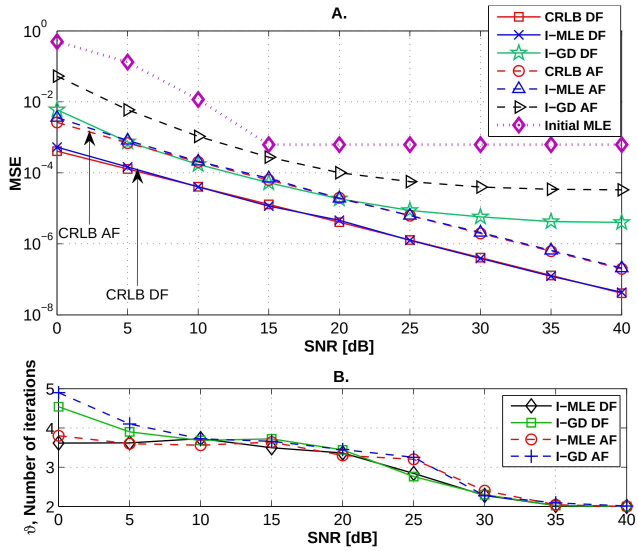

Fig. 4. The MSE of I-MLE, I-GD, and the initial MLE for the estimation of $\tau_1^{[\mathrm{DF}]}$ for **DF networks** VS. the CRLB in (6) with $R = 4$, $L = 64$, and $N = 2$.

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