# Bandwidth Efficient Channel Estimation for Full Duplex Communication Systems

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***Abstract*—In this paper, the channel estimation problem in full duplex (FD) point-to-point wireless communication systems is investigated. Because of the existence of two interfering and communication channels, this problem is fundamentally different from the conventional channel estimation problems in half duplex (HD) communication systems. Here, we propose a blind channel estimator that simultaneously estimates the channel parameters of the FD system without requiring time division duplex. A major problem for blind estimators is the identifiability of the parameters. Therefore, thorough analysis is presented to determine the identifiable parameters in this estimation problem. It will be shown that a completely blind estimator utilizing a symmetric modulation set suffers from phase ambiguity. For this reason, only the channel magnitudes can be estimated blindly. It is further presented that if the modulation set is not centered around the origin then this phase ambiguity can be resolved.**

## I. Introduction

Wireless communication devices are traditionally designed for half duplex (HD) operation, where they can transmit and receive data on two different frequency bands. However this operation mode fundamentally limits the bandwidth efficiency of the communicating nodes. Therefore, a significant trend of research is now directed at wireless devices that are capable of transmission and reception in the same frequency band. These devices that are known as full duplex (FD) wireless devices, are more bandwidth efficient than their HD counterparts [1]–[3]. An important part of the research in this area is dedicated to the theoretical analysis of the performance of these devices. More specifically, the data rate improvements over the HD operation is analyzed in [4]–[6]. These studies reveal that a major flaw that severely affects the performance of FD devices, is self-interference caused by the shared channel for transmission and reception. To overcome this issue, a major research stream has emerged to deal with completely cancelling or partially removing the self-interference.

Active and passive cancellation techniques are proposed in the literature to address this issue. In passive cancellation transmitting and receiving antennas are well isolated to reduce the amount of interference. On the other hand, active cancellation techniques are proposed alongside passive cancellation to achieve better performance by further mitigating the self-interference. For efficient active self-interference cancellation the knowledge of the channel is essential. Therefore, accurate channel estimation is crucial to realizing the potential of FD devices. The problem of channel estimation in FD point-to-point systems is different from that of classical HD systems as in these systems there exist two channels, i.e, the self interference channel and the communication channel. These channels need to be simultaneously estimated at the receiver end. The self-interfering channel should be estimated for self-interference cancelation and the communication channel is required for data detection at the receiver.

The current approach for channel estimation is to keep the transmitting nodes silent while estimating the self interference channel. Subsequently, pilots are sent for the estimation of the channel parameters [7]–[10]. In [7], five pilots are sent to estimate the interfering channel parameters while the transmitting node is kept silent. As a result, in total, ten pilots are transmitted to estimate both the interfering and communication channels. Thus, such a channel estimation approach reduces bandwidth efficiency.

In this paper we present a blind channel estimator for FD wireless communication devices that does not require pilot training and enhances the bandwidth efficiency of the FD system. It is assumed that full knowledge of the distribution of transmitted symbols is available. With this assumption, it will be shown that the blind channel estimation problem for this system requires solving a non-convex Gaussian Mixture maximization that can be solved using the Expectation Maximization (EM) approach. Unlike existing work in the literature that focus on blind estimating of multiple channel parameters

[11]–[13], the identifiability condition of the stochastic blind channel estimation for FD systems using M-PSK modulations is fully analyzed. This analysis shows a blind channel estimator with symmetric modulation set suffers from phase ambiguity. Consequently, we propose a fully blind channel estimator to estimate the magnitudes and the phases of the channels using a shifted (asymmetric) modulation set. The proposed estimator using the asymmetric modulation set allows for fully blind estimation of the channels parameter without any pilot training. As opposed to other similar works [11], [12], [14], [15] where pilot training is required to resolve the inherent phase ambiguity of blind estimators, the proposed algorithm resolves this issue by shifting the modulation set and normalizing the energy . Consequently, it allows for fully blind estimation with no training. For the same number of data samples and pilots, it is evident by the simulation results that the MSE performance of the proposed estimator is not very far away from its pilot based counterpart.

### *A. Paper Organization*

The rest of the paper is organized as follow, in Section II, we present the system model and formulate the channel estimation problem. The identifiability problem and the blind estimator are discussed in Sections III. This is followed by Section IV, where the simulation results are reported. Finally in Section V, we conclude the paper.

### *B. Notation*

Bold face small letters, e.g., $\mathbf{x}$, are used for vectors, bold face capital letters, e.g., $\mathbf{X}$, are used for matrices and capital letters, e.g. $\mathcal{A}$, are used for sets. $|.|$ denotes the absolute value operator and $E[\boldsymbol{.}]$ represents the expected values of the argument. The real and imaginary parts of a complex quantity are represented by $\Re\{\boldsymbol{.}\}$ and $\Im\{\boldsymbol{.}\}$, respectively. $\mathcal{CN}(\mu, \sigma^2)$ denotes a complex Gaussian distribution with mean $\mu$ and variance $\sigma^2$. $f(y)$ is used to denote the probability distribution of $y$, $c$ represents a constant scalar and $e^{\mathrm{j}\theta} = \cos\theta + \mathrm{j}\sin\theta$. Finally, $I_{\{x_k\}}(x)$ is the indicator function, $I_{\{x_k\}}(x) = \begin{cases} 1 & x = x_k \\ 0 & x \neq x_k \end{cases}$

## II. System Model

We consider a point-to-point Full Duplex (FD) communication system as shown in Fig.1. Nodes $a$ and $b$ are equipped with a single antenna for reception and transmission. It is assumed that both nodes operate in FD mode, i.e., they transmit and receive simultaneously. All the channels are assumed to be Rayleigh flat fading channels with mean zero and unit variance, i.e., $\mathcal{CN}(0, 1)$. We further assume that the channels remain constant for the duration of estimation. M-PSK modulation is used at both nodes to transmit the data over the channel, it is also assumed that all the symbols in the modulation set are equiprobable. Let us denote the M-PSK modulated vector transmitted from node $a$ and $b$ by $\mathbf{x_a}$ and $\mathbf{x_b}$, respectively. Then the received vectors at nodes $a$ and $b$ are given by

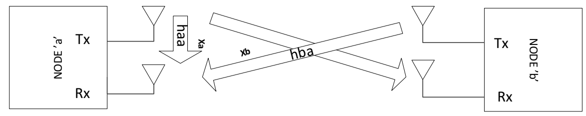


Fig. 1. Full Duplex Point to Point System with single transmit and receive antenna

$$\mathbf{y_a} = h_{aa}\mathbf{x_a} + h_{ba}\mathbf{x_b} + \mathbf{w_a} \quad (1a)$$

$$\mathbf{y_b} = h_{ba}\mathbf{x_a} + h_{bb}\mathbf{x_b} + \mathbf{w_b} \quad (1b)$$

where $h_{aa}$ and $h_{bb}$ are the self interference channels of node $a$ and $b$ respectively. These channels are created because of simultaneous transmission and reception in full duplex communication. On the other hand, $h_{ba}$ represents the reciprocal communication channel between nodes $a$ and $b$. The channel estimation problem is the same for both nodes $a$ and $b$. Therefore, without loss of generality, we will only consider the channel estimation problem at node $a$. The aim is to estimate the self-interfering channel $h_{aa}$ and the communication channel $h_{ba}$ at node $a$.

### *A. The Channel Estimation Problem*

The channel estimation problem at node $a$ is to estimate the parameters of the probability distribution function (PDF) of the observation vector $\mathbf{y_a}$. Hence, the desired parameters are $h_{aa}$ the self interference channel and $h_{ba}$ the communication channel. Knowing the self-interfering symbols, i.e. $\mathbf{x_a}$, we assume a discrete uniform distribution for the communication symbols, i.e. $\mathbf{x_b}$, and complex Gaussian PDF with mean zero and unit variance for one component of noise vector $w_i$. Therefore, for the conditional PDF of only one observation $y_{ai}$ we have

$$y_{a_i} = h_{aa}x_{a_i} + h_{ba}x_{b_i} + w_i : for(i = 1, 2, \cdots, N) \quad (2)$$

$$f(y_{a_i}|x_{b_i} = x_k) = \frac{1}{\pi\sigma^2}\exp\left(\frac{-1}{\sigma^2}|y_{a_i} - h_{ba}x_k - h_{aa}x_{a_i}|^2\right) \quad (3)$$

where in (3), $x_k = \sqrt{P}e^{\frac{\mathrm{j}(2k-1)\pi}{M}}$ is the $k^{th}$ symbol drawn from the M-PSK modulation set $\mathcal{A} = \{x_1, x_2, ..., x_M\}$ with probability $P(x_{b_i} = x_k) = \frac{1}{M}$, $P$ is the transmit power and $N$ is the number of observations. Then the marginal PDF of one observation is given by

$$f(y_{a_i}) = \sum_{k=1}^{M} f(y_{ai}|x_{b_i} = x_k)P(x_{b_i} = x_k) \tag{4}$$
$$= \frac{1}{M\pi\sigma^2}\sum_{k=1}^{M}\exp\left(\frac{-1}{\sigma^2}|y_{a_i} - h_{ba}x_k - h_{aa}x_{a_i}|^2\right)$$

Therefore for the joint PDF of the observations we have:

$$f(\mathbf{y_a}; h_{aa}, h_{ba}) = \prod_{i=1}^{N} f(y_{a_i}) = \left(\frac{1}{M\pi\sigma^2}\right)^N \tag{5}$$
$$\prod_{i=1}^{N}\sum_{k=1}^{M}\exp\left(\frac{-1}{\sigma^2}|y_{a_i} - h_{ba}x_k - h_{aa}x_{a_i}|^2\right)$$

Accordingly, the log-likelihood function is given by:

$$\mathcal{L}(h_{aa}, h_{ba}) = \ln\prod_{i=1}^{N} f(y_{a_i}) \tag{6}$$
$$= -N\ln(M\pi\sigma^2) +$$
$$\sum_{i=1}^{N}\ln\left(\sum_{k=1}^{M}\exp\left(\frac{-1}{\sigma^2}|y_{a_i} - h_{ba}x_k - h_{aa}x_{a_i}|^2\right)\right)$$

Finally, the ML solution is:

$$[\tilde{h}_{aa}, \tilde{h}_{ba}] = \arg\max_{h_{aa}, h_{ba}} \mathcal{L}(h_{aa}, h_{ba}) \tag{7}$$

The likelihood function given by (6), is a Gaussian Mixture function and a non-convex function on the desired parameters. This non-convexity is the result of assuming a discrete uniform distribution for unknown vector $\mathbf{x_b}$. Therefore, we need to resort to numerical algorithms like the EM algorithm to solve (7).

## III. Blind Estimation Using Expectation Maximization

### *A. The identifiability problem of completely blind channel estimation*

If $\mathbf{y}$ is a random variable distributed according to $f(\mathbf{y}; \theta)$, then $\theta$ is said to be unidentifiable on the basis of $\mathbf{y}$, if for all $\mathbf{y}$ there exists $\theta \neq \theta'$ for which $f(\mathbf{y}; \theta) = f(\mathbf{y}; \theta')$ and if $\theta$ is unidentifiable, it is impossible to estimate $\theta$ [16].

*Theorem:* The parameter $h_{ba}$ is an un-identifiable parameter of the PDF of observations in (3) if and only if the modulation set is symmetric.[1]

*Proof:* We assume $h_{ba} \neq h_{ba}^*$ and for simplicity we define $\alpha_{i,k}(y_{a_i}) = \exp\left(\frac{-1}{\sigma^2}|y_{a_i} - h_{ba}x_k - h_{aa}x_{a_i}|^2\right)$, then, we can write (5) as follow:

$$f(\mathbf{y}; h_{aa}, h_{ba}) = \left(\frac{1}{\pi\sigma^2 M}\right)^N \prod_{i=1}^{N}\sum_{k=1}^{M}\alpha_{i,k}(y_{a_i}) \tag{8}$$
$$= \beta_1(y_{a_1})\cdots\beta_N(y_{a_N})$$

Where in (8), $\beta_i(y_{a_i}) = \sum_{k=1}^{M}\alpha_{i,k}(y_{a_i})$. Similarly for $h_{ba}^*$ we can have the following

$$f(\mathbf{y}; h_{aa}, h_{ba}^*) = \left(\frac{1}{\pi\sigma^2 M}\right)^N \prod_{i=1}^{N}\sum_{k=1}^{M}\alpha'_{i,k}(y_{a_i}) \tag{9}$$
$$= \beta'_1(y_{a_1})\cdots\beta'_N(y_{a_N})$$

It is clear that $\alpha_{i,k}(y_{a_i})$ and $\beta_i(y_{a_i})$ are independent functions of $y_{a_i}$. Therefore, we can say that if $f(\mathbf{y}; h_{aa}, h_{ba}) = f(\mathbf{y}; h_{aa}, h_{ba}^*)$ then $\beta_i(y_{a_i}) = \beta'_i(y_{a_i})$ and consequently there exists a permutation $\sigma$ on $k = 1, 2, \cdots, M$ for which:

$$\alpha_{i,k}(y_{a_i}) = \alpha_{i,\sigma(k)}(y_{a_i})$$

For simplicity, we define $\gamma(y_{a_i}) = y_{a_i} - h_{aa}x_{a_i}$

$$|\gamma(y_{a_i}) - h_{ba}x_k|^2 = |\gamma(y_{a_i}) - h_{ba}^* x_\sigma(k)|^2 \tag{10}$$

For (10) to hold for all $y_{a_i}$, we need to have $h_{ba}x_k = h_{ba}^* x_{\sigma(k)}$ for $k = 1, 2, \cdots, M$. This will give us $M$ different equations which should hold concurrently to have an un-identifiable problem. Therefore, the desired parameters are un-identifiable if and only if there exists a permutation $\sigma$ such that $\frac{x_k}{x_{\sigma(k)}} = c$ for $k = 1, 2, \cdots, M$. Lets define $m = \sigma(k)$, then

$$c = \frac{x_k}{x_m} = \frac{\sqrt{P}e^{\frac{\mathrm{j}(2k-1)\pi}{M}}}{\sqrt{P}e^{\frac{\mathrm{j}(2m-1)\pi}{M}}} = e^{\frac{\mathrm{j}(k-m)(2\pi)}{M}} \tag{11}$$

Consequently, we can say that the un-identifiability problem happens if $l = k - m$ is a constant. Hence, for a given $h_{ba}$, $M$ different $h_{ba}^{*,l}$ exist that $h_{ba} = e^{\mathrm{j}l}h_{ba}^{*,l}$, which generate the same PDF of observations[2] and consequently can not be identified from the desired parameter. However, if the modulation set is shifted then no permutation can be found for which $x_k = cx_{\sigma(k)}$.

### *B. Channel Estimation using shifted (asymmetric) modulation set:*

Since for an asymmetric modulation set a permutation $\sigma$ such that $\frac{x_k}{x_{\sigma(k)}} = c$ does not exist, $h_{ba}$ is identifiable. An asymmetric modulation set can be achieved by shifting the modulation set by a certain value, so instead of using $\mathcal{A} = \{x_1, x_2, ..., x_M\}$, we can use the modulation

[1] $\mathcal{A}$ is a symmetric modulation set if there exists a permutation $\sigma$ of $\{1, 2, ..., M\}$ and a constant $c$ such that for all $k : x_k = cx_{\sigma(k)}$

[2] $\angle\frac{\mathrm{j}l(2\pi)}{M}$ has $M$ different value in $(0, 2\pi)$

| | |
|---|---|
| Number of observations $N$ | 8 |
| Modulation | QPSK |
| SNR range | 0 to $30dB$ |
| Number of Pilots | 8 |
| Number of Monte Carlo Points | 1000 |

TABLE I
SIMULATION PARAMETERS

set $\mathcal{A}' = \{x'_1, x'_2, ..., x'_M\} = \{x_1, x_2, ..., x_M\} + \gamma$. We can then normalize the set $\mathcal{A}'$ to ensure its average energy does not exceed the average energy of modulation set $\mathcal{A}$. For asymmetric modulation set the EM algorithm's steps are derived in Appendix A and are given by

- **Expectation Step:**

$$Q(\boldsymbol{\theta}|\boldsymbol{\theta}^{(n)}) = -N \ln(M\pi\sigma^2) - \frac{1}{\sigma^2}\sum_{i=1}^{N}\sum_{k=1}^{M} T_{j,i}^{(n)} |y_{a_i} - h_{ba}x'_k - h_{aa}x_{a_i}|^2 \quad (12)$$

- **Maximization Step:**

$$\boldsymbol{\theta}^{(n+1)} = \arg\max_{\boldsymbol{\theta}} Q(\boldsymbol{\theta}|\boldsymbol{\theta}^{(n)}) = \arg\min_{\boldsymbol{\theta}} \sum_{i=1}^{N}\sum_{k=1}^{M} T_{k,i}^{(n)} |y_{a_i} - h_{ba}x'_k - h_{aa}x_{a_i}|^2 \quad (13)$$

Where $T_{k,i}^{(n)}$ is defined as follow

$$T_{k,i}^{(n)} = \frac{\exp\left(\frac{-1}{\sigma^2}|y_{a_i} - h_{ba}^{(n)}x'_k - h_{aa}^{(n)}x_{a_i}|^2\right)}{\sum_{k'=1}^{M}\exp\left(\frac{-1}{\sigma^2}|y_{a_i} - h_{ba}^{(n)}x'_{k'} - h_{aa}^{(n)}x_{a_i}|^2\right)} \quad (14)$$

## IV. SIMULATION RESULTS

For the blind estimator proposed in this paper, we simulated the average MSE and obtained the results for different signal to noise ratios. The simulations parameters are tabulated in the Table I. The performance of the blind estimator for the shifted modulation set is presented in Fig.2 . In this case, all the channels are identifiable. The result is obtained for the shift of $\gamma = 1$. This result shows that the performances of the blind estimators are not very far away from the pilot based estimators.It is worth noting that once orthogonal piloting is used, then the channel estimation problems becomes an estimation problem for parameters in a linear Gaussian model. The MLE solution in this case is Minimum Variance Unbiased (MVU) estimator and it reaches Cramer-Rao-Bound (CRB) [17]. The effect of different shifts on estimation of the channels was also investigated. The average MSE for $SNR = 10dB$ and different values of shift is shown Fig.3. The results shows that minimum MSE is produced when $\gamma = -1, 1$.

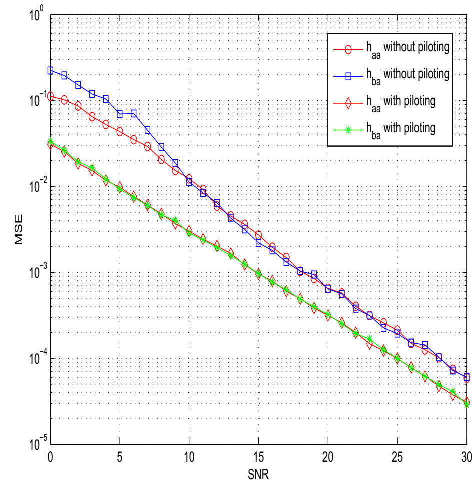


Fig. 2. Average MSE for $h_{ba}$ and $h_{aa}$ for shifted modulation

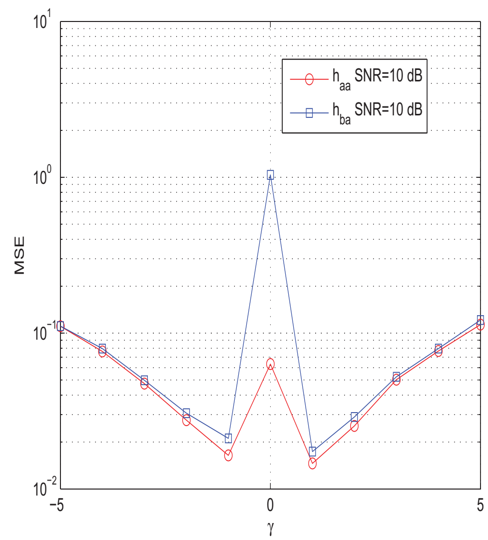


Fig. 3. Effect of different shifts on estimation

## V. CONCLUSION

In this paper, we investigated the problem of the channel estimation for FD wireless systems. We proposed a blind channel estimator using shifted modulation set and the EM algorithm to estimate the interfering and communication channels simultaneously. The identifiability problem of the blind estimator was thoroughly investigated to determine the identifiable channels. It was shown that for a symmetric modulation set channel $h_{ba}$ is not identifiable. However, a shift in a modulation set will make both $h_{aa}$ and $h_{ba}$ identifiable. The simulation results show that with asymmetric modulation set the MSE performance of the estimators is not very far from the pilot based estimators. It was also shown through simulations that a shift of $\gamma = -1, 1$ produces minimum MSE.

## Appendix A
## EM-Algorithm

In this section we derive the equations for the EM algorithms which are used to find the MLE solution of (7). We start with the PDF of one observation, the joint PDF for one observation and $x_{b_i}$ is given by

$$f(y_{a_i}, x_{b_i}; \boldsymbol{\theta}) = \frac{1}{M\pi\sigma^2} \sum_{k=1}^{M} I_{\{x'_k\}}(x_{b_i}) \exp\left(\frac{-1}{\sigma^2}|y_{a_i} - h_{ba}x'_k - h_{aa}x_{a_i}|^2\right) \quad \text{(A.1)}$$

Taking the natural logarithm from the above we will have

$$\ln(f(y_{a_i}, x_{b_i}; \boldsymbol{\theta})) = -\ln(M\pi\sigma^2) - \frac{1}{\sigma^2}\sum_{k=1}^{M} I_{\{x'_k\}}(x_{b_i})|y_{a_i} - h_{ba}x'_k - h_{aa}x_{a_i}|^2 \quad \text{(A.2)}$$

Finally, for the joint PDF of all the observations we have

$$\ln(f(y_a, x_b; \boldsymbol{\theta})) = \sum_{i=1}^{N} \ln(f(y_{a_i}, x_{b_i}; \boldsymbol{\theta})) \quad \text{(A.3)}$$
$$= -N\ln(M\pi\sigma^2) - \frac{1}{\sigma^2}\sum_{i=1}^{N}\sum_{k=1}^{M} I_{\{x'_j\}}(x_{b_i})|y_{a_i} - h_{ba}x'_k - h_{aa}x_{a_i}|^2$$

To find the $Q(\boldsymbol{\theta}|\boldsymbol{\theta}^{(n)})$ for the expectation step we need to find $E_{X_b|y;\boldsymbol{\theta}^{(n)}}[\log f(y, x_b|\boldsymbol{\theta})]$ as follow

$$Q(\boldsymbol{\theta}|\boldsymbol{\theta}^{(n)}) = E_{x_b|y;\boldsymbol{\theta}^{(n)}}[\log f(y, x_b; \boldsymbol{\theta})] \quad \text{(A.4)}$$
$$= -N\ln(M\pi\sigma^2) - \frac{1}{\sigma^2}\sum_{i=1}^{N} \sum_{k=1}^{M} E_{x_b|y;\boldsymbol{\theta}^{(n)}}[I_{\{x'_k\}}(x_{b_i})]|y_{a_i} - h_{ba}x'_k - h_{aa}x_{a_i}|^2$$

We also define a new function $T_{j,i}^{(n)}$ given by

$$T_{k,i}^{(n)} = E_{x_b|y;\boldsymbol{\theta}^{(n)}}[I_{\{x'_k\}}(x_{b_i})]$$
$$= P(x_{b_i} = x'_k | y_a, \boldsymbol{\theta}^{(n)})$$
$$= \frac{\exp\left(\frac{-1}{\sigma^2}|y_{a_i} - h_{ba}^{(n)}x'_k - h_{aa}^{(n)}x_{a_i}|^2\right)}{\sum_{k'=1}^{M}\exp\left(\frac{-1}{\sigma^2}|y_{a_i} - h_{ba}^{(n)}x'_{k'} - h_{aa}^{(n)}x_{a_i}|^2\right)}$$

Finally, $Q(\boldsymbol{\theta}|\boldsymbol{\theta}^{(n)})$ can be rewritten as

$$Q(\boldsymbol{\theta}|\boldsymbol{\theta}^{(n)}) = -N\ln(M\pi\sigma^2) - \frac{1}{\sigma^2}\sum_{i=1}^{N}\sum_{j=1}^{M} T_{k,i}^{(n)}|y_{a_i} - h_{ba}x'_k - h_{aa}x_{a_i}|^2 \quad \text{(A.5)}$$

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