

# Gain Optimization and Direction of Arrival Estimation For ESPAR Antenna Using Trust Region Reflective Algorithm

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**Abstract**—Paper encompass numerical analysis techniques for Direction of Arrival (DOA) estimation in seven element Electronically Steerable Parasitic Array Radiator (ESPAR) antenna. Power Method is proposed for DOA estimation, in this method Direction of Arrival (DOA) is estimated by steering the beam and measuring the received signal strength. ESPAR antenna is simulated at 2.45 GHz, there is an active central element which is surrounded by an array of six parasitic elements, each parasitic element are placed at  $\lambda/4$  distance from central active element. Parasitic element are loaded with variable reactance such that beam can be steered in continuous  $360^\circ$  by varying different values of reactance. Traditional direction of arrival algorithms i.e Multiple Signal Classification (MUSIC) and Estimation of Signal Parameters via Rotation Invariance Techniques (ESPRIT) involve multiple feed therefore they can not be applied directly on ESPAR antenna as ESPAR antenna has single feed for each beam. We have proposed a Power Method for DOA estimation, where the reactance matrix for placing maxima at a given value of azimuth angle( $\phi$ ) is calculated by using quasi Newton method. Results are later compared with the Finite Element Method (FEM) and Method of Moments (MoM).

**Index Terms**—parasitic, re-configurable, beam steering, numerical analysis .

## I. INTRODUCTION

SMART antennas have capability of adaptive beam forming, where the beam can be steered by way of mechanical beam steering (radars etc) and electronic beam steering. Due to presence of moving parts mechanical beam steering systems involve wear and tear. Moreover these systems also require higher power consumption to drive motors and other mechanical devices. Electronic beam steering is achieved via phased array antennas [1]. Phased array antennas are very expensive to implement since each element of antenna array has to be separated by atleast half of wavelength more over separate phase and excitation is required at each element therefore overall phased array antennas are expensive to implement and have relatively larger size.

If two antenna elements are placed closer to each other phenomena of mutual coupling will occur, ESPAR antennas

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use advantage of mutual coupling for changing direction of beam therefore ESPAR antennas are cost effective and required far less space as compared to phased array antennas antennas[3]. Traditional ESPAR antennas have one central active element and it is surrounded by an array of passive elements, excitation is provided to central element only. Introduction of 4G has led to extensive research towards 5G systems. 5G systems involve machine to machine, machine to man and man to man communication. There is a need to incorporate smart antenna systems to handle high data rate requirements in future communication systems. It is estimated that by 2020 more than 55 percent of all communication taking place will involve wireless communication. It is desired to minimize interference, increase data rates, improve encryption over wireless links and reduce the unwanted effects of continuous stray radiation on human health. With the advent of 5G and IoT, ESPAR antenna have gained importance and they offer economical as well as space efficient solution for next generation communication networks. Common techniques to increase data rate and make efficient use of allocated spectrum have approached limits. In accordance with Shannons equation the lesser the interference more will be the channel bandwidth.

$$C = B * \log_2(1 + (S/N)) \quad (1)$$

Here C is the channel Capacity, B is the channel bandwidth , (S/N) is signal to noise ratio, form the above mentioned equation it is evident that more the signal to noise ratio more is the channel capacity. Signal to Interference Noise Ratio (SINR) for Wireless Local Area Network (WLAN) systems and cellular networks can be improved by using adaptive antenna systems. Using ESPAR antennas overall datarate can be increased and as SNR is increased.

Paper includes mathematical modeling and numerical analysis techniques for calculation of constrained reactances set for gain optimization and direction of arrival estimation in ESPAR antenna, derived results are compared with Finite Element Method and Method of Moments.

## II. CONCEPT OF ESPAR ANTENNA

Conventional ESPAR antennas have one active element in center which is surrounded by a circular array of parasitic elements, parasitic are loaded with varactor diode . All elements are cylindrical in nature in traditional ESPAR antenna, contrary in our designed antenna we have made center element conical, due to conical nature of central element antenna is

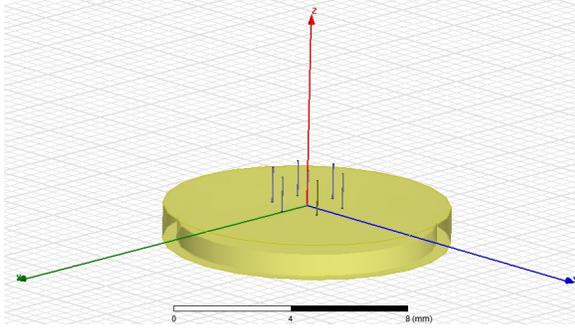


Fig. 1. Simulated Traditional ESPAR Antenna.

made more broad band as it resonates over increased range of frequencies. Fig. 1 shows design of ESPAR antenna with 1 active element[3].

### III. MATHEMATICAL MODELING

In circular array antenna transmitted electric field from active element induces current and voltage in surrounding parasitic elements. Voltage and Current Vectors are represented as

$$I = [i_o \ i_1 \ i_2 \ i_3 \ i_4 \ i_5 \ i_6]^T \quad (2)$$

$$V = [v_o \ v_1 \ v_2 \ v_3 \ v_4 \ v_5 \ v_6]^T \quad (3)$$

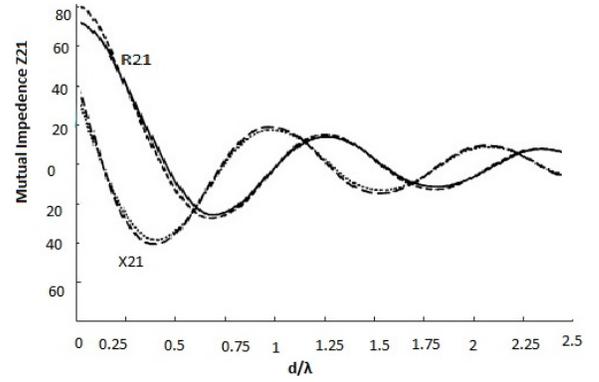
Current and voltages induced depend on value of reactance matrix and vary with changing reactance matrix therefore Vector Effective Length Equivalent Weight Vector (VEL-EWV) method is used to compute induced current and electric field[5]. Using VEL-EWV method Electrical field is measured terms of theta ( $\theta$ ) and Phi ( $\phi$ ). Electric field is product of induced current and vector effective length, vector effective length is dependent upon reactance matrix and varies with change in reactance matrix which in turn changes Electric Filed thus beam is steered.

$$E(\theta, \phi) = -j \frac{Z_0}{2\pi} \frac{e^{-jkr}}{r} \sum_{m=0}^M l_m(\theta) i_m a_m(\theta, \phi) \quad (4)$$

Here m varies from 0 to M, 0 is index value for feed element and M is total number of feed elements in each circular array, M=6 in our case.  $i_m$  port current for mth element,  $l_m$  is vector effective length,  $Z_0$  is characteristic impedance,  $\lambda$  is wavelength and  $a_m(\theta, \phi)$  depicts  $\theta$  and  $\phi$  components of steering vector[4]. Steering vector  $a_m$  is defined as

$$a_m = e^{jk(\sin(\theta)\cos(\phi)x_m + \sin(\theta)\sin(\phi)y_m + \cos(\theta)z_m)} \quad (5)$$

Beam is steered by mutual coupling between parasitic elements, admittance matrix will give mutual coupling between each element[5,6]. Mutual coupling between elements is as factor of distance Fig.3 shows  $Z_{21}$  between dipoles placed side by side as a function of distance between them. Admittance matrix for circular array is given by

Fig. 2.  $Z_{21}$  Vs Distance between Elements Placed Side by Side.

$$Y = \begin{bmatrix} y_{00} & y_{01} & y_{02} & y_{03} & y_{04} & y_{05} & y_{06} \\ y_{01} & y_{11} & y_{12} & y_{13} & y_{14} & y_{15} & y_{16} \\ y_{02} & y_{21} & y_{22} & y_{23} & y_{24} & y_{25} & y_{26} \\ y_{03} & y_{31} & y_{32} & y_{33} & y_{34} & y_{35} & y_{36} \\ y_{04} & y_{41} & y_{42} & y_{43} & y_{44} & y_{45} & y_{46} \\ y_{05} & y_{51} & y_{52} & y_{53} & y_{54} & y_{55} & y_{56} \\ y_{06} & y_{61} & y_{62} & y_{63} & y_{64} & y_{65} & y_{66} \end{bmatrix} \quad (6)$$

In this case we have circular array with each parasitic element at distance of  $\lambda/4$  from center therefore we have

$$y_{13} = y_{24} = y_{35} = y_{46} = y_{51} = y_{62}$$

$$y_{01} = y_{02} = y_{03} = y_{04} = y_{05} = y_{06}$$

$$y_{12} = y_{23} = y_{34} = y_{45} = y_{56} = y_{61}$$

$$y_{11} = y_{22} = y_{33} = y_{44} = y_{55} = y_{66}$$

$$y_{14} = y_{25} = y_{36}$$

Induced current is given as

$$I = \begin{bmatrix} y_{00} & y_{01} & y_{02} & y_{03} & y_{04} & y_{05} & y_{06} \\ y_{01} & y_{11} & y_{12} & y_{13} & y_{14} & y_{15} & y_{16} \\ y_{02} & y_{21} & y_{22} & y_{23} & y_{24} & y_{25} & y_{26} \\ y_{03} & y_{31} & y_{32} & y_{33} & y_{34} & y_{35} & y_{36} \\ y_{04} & y_{41} & y_{42} & y_{43} & y_{44} & y_{45} & y_{46} \\ y_{05} & y_{51} & y_{52} & y_{53} & y_{54} & y_{55} & y_{56} \\ y_{06} & y_{61} & y_{62} & y_{63} & y_{64} & y_{65} & y_{66} \end{bmatrix} V \quad (7)$$

Port current  $i_m$  is given as

$$[I_m] = ([Z_{mn}] + [X_m])^{-1} [V_s] \quad (8)$$

$$[X_m] = \text{diag}[z_0, jx_1, jx_2, jx_3, jx_4, jx_5, jx_6] \quad (9)$$

$$[V_s] = [v_s, 0, 0, 0, 0, 0, 0]^T \quad (10)$$

where  $Z_{mn}$  is impedance between mth and nth element, it is inverse of  $Y_{mn}$ .  $Y_{mn}$  is given in (6).  $v_s$  is open circuit voltage,  $z_0$  is initial impedance of feed element. Vector effective length used in (4) for far field calculation is

$$l_m = l_m^0 (1 - \alpha_m x_m) \quad (11)$$

here  $l_m^0$  and  $\alpha_m$  are structural parameters [2], where as  $x_m$  is reactance set of parasitic elements, m varies from 1 to 6 for 6 parasitic elements[7].  $l_0^0$  is calculated as

$$l_0 = l_0^0 (1 - j\alpha_o Z_{in}) \quad (12)$$

Overall directivity in given direction of  $\theta$  and  $\phi$  is

$$D(\theta, \phi) = 4\pi \frac{|E\theta, \phi|^2}{Z_o |v_o|^2 \text{Re}(\frac{1}{Z_{in}})} \quad (13)$$

Simplifying (13) using (4) yields

$$D(\theta, \phi) = \frac{\pi Z_0 \sin\theta |\sum_{m=0}^M l_m i_m |_{Z_s=0} a_m(\theta, \phi)|^2}{\lambda^2 |v_s|^2 \text{Re}(\frac{1}{Z_{in}})} \quad (14)$$

$$S_{11} = \frac{Z_{in} - Z_s}{Z_{in} + Z_s} \quad (15)$$

Using 14 and 15 overall gain of each circular array in given direction of  $\theta$  and  $\phi$  is

$$G(\theta, \phi) = (1 - |S_{11}|^2) D(\theta, \phi) \quad (16)$$

#### IV. ALGORITHM FOR GAIN OPTIMIZATION AND DOA ESTIMATION

Gain of each array is given by (16), from (16), (8) and (5) it is shown that gain is dependent on values of reactance matrix  $X_m$ . There are six variables i.e  $x_1, x_2, x_3, x_4, x_5, x_6$  and we have to calculate their value such that gain is maximized. Finite Element Method is given in used in [4], in Finite Element Method problem is divided into small uniform geometries called finite elements. Equations are solved for finite elements, results of equations are later assembled to form larger system of equation. Instead of using finite element method we have proposed a numerical analysis technique in which electric field is calculated on basis of vector effective length equivalent weight vectors (VEL-WEF). Using constrained nonlinear optimization algorithm value of reactance matrix is calculated for placing maxima in given direction[7]. Constrained nonlinear optimization algorithm given minima of  $1/G(\theta, \phi)$  function for constraint value of reactance matrix.  $X_0$  is the initial matrix input for reactance matrix. We have initialized reactance matrix as  $X$  (17) and the algorithm give value of minimum of  $1/G(\theta, \phi)$  such that final reactance matrix is within bounds mentioned in (19)

$$X_0 = [0 \ 0 \ 0 \ 0 \ 0 \ 0] \quad (17)$$

$$A = [-300 \ -100 \ -100 \ -200 \ -200 \ -300], \quad b = [-100] \quad (18)$$

$$A \cdot X_o \leq b \quad (19)$$

In constrained nonlinear optimization algorithm A and b set the constraint on final value of reactance matrix X so that we can use varactor diodes available in market to yield desired reactance set. In constrained nonlinear optimization algorithm quadratic model at point  $x_k$  is given as

$$m_k(p) = f(x_k) + \nabla f(x_k)^T p + \frac{1}{2} p^T B_k p \quad (20)$$

$B_k$  is Hessian matrix it is updated after each iteration, we get search direction  $p_k$  by

$$p_k = -B_k^{-1} \nabla f(x_k) \quad (21)$$

From  $p_k$  we get next point i.e  $x_{k+1}$

$$x_{k+1} = x_k + \alpha_k p_k \quad (22)$$

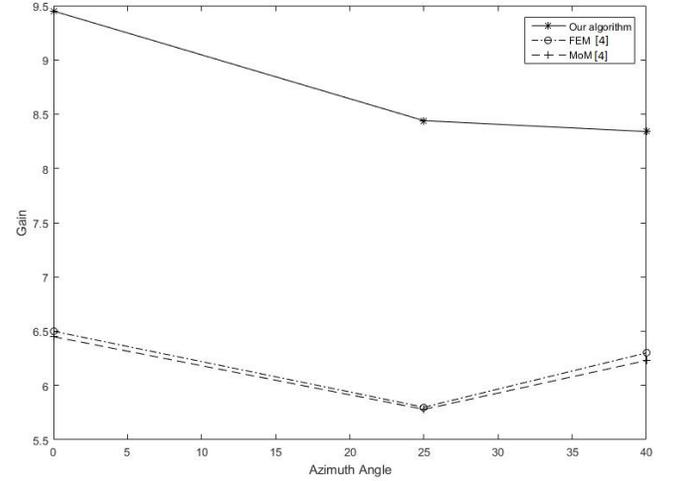


Fig. 3. Gain Comparison with FEM and MoM.

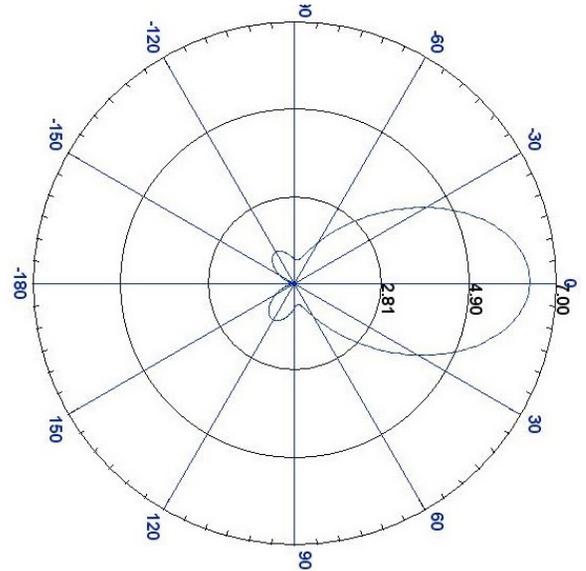


Fig. 4. Z with Finite Element Method.

We have used built in function of Matlab, which uses constrained nonlinear optimization algorithm for calculating values of reactance matrix that yields minima at  $1/G(\theta, \phi)$ , minima of  $1/G(\theta, \phi)$  corresponds to maxima of  $G(\theta, \phi)$ . Finite Element gives gain of 6.5 dB [4] with the reactance set in table I. Fig. (4) shows gain plot at  $\phi=0^\circ$  using Finite Element Method. Our algorithm has optimized gain by 2.9 dB table II shows optimized reactance set with gain of 9.45 dB. Comparison of our method with FEM and MoM is given in Fig.4

For the direction of  $0^\circ$  algorithm has taken 905 iterations, which is more than than quasi newton unconstrained method

TABLE I  
REACTANCE MATRIX WITH FINITE ELEMENT METHOD

$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$
-150	250	78	26	78	250

TABLE II  
REACTANCE MATRIX WITH OPTIMIZATION ALGORITHM

X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>	X <sub>4</sub>	X <sub>5</sub>	X <sub>6</sub>
-570500	6354900	0	0	0	55480

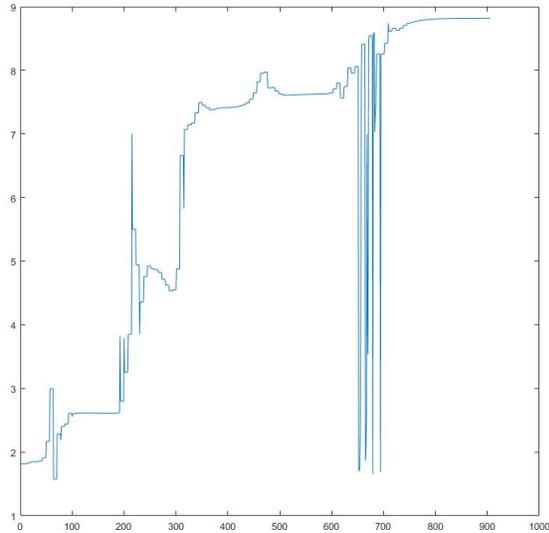


Fig. 5. Step-wise Gain Convergence.

which converges in 898. This algorithm has taken more iterations due to the fact that reactance matrix is constrained. Fig. (5) shows convergence of gain to 9.45.

For direction of arrival estimation we are using Power Method, in this method initially all 360 reactance sets are loaded one after another to steer beam in 360°. Received signal strength is continuously measured for each reactance set using digital to analog converter, initial direction is locked as direction with maximum received signal strength, upon reducing is received signal strength from threshold value beam is first rotated towards right by 1°. If received signal strength is dropped beam is rotated towards left by 2°. Hence active source is constantly tracked and direction of arrival is reflected in terms of pre calculated reactance matrix. Fig. (6) shows scenario for DOA estimation using power method.

## V. CONCLUSION

With the advent of 5G and IoT there is immense potential for utility of beam steering antenna in order to enhance data rate and minimize interference. For higher data-rates dual beam re-configurable antenna with single feed is designed. Each beam is steered by using varying reactance set. Vector Effective Length Equivalent Weight Method is used for mathematical modeling.

Mathematical model is optimized for best gain using VEL-EWV and constrained nonlinear optimization algorithm. After optimization gain on 9.45 dB is achieved which is 2.95 dB higher than the gain calculated by Finite Element Method.

For Direction of Arrival estimation power method is used in this method active source is constantly transmitting signal

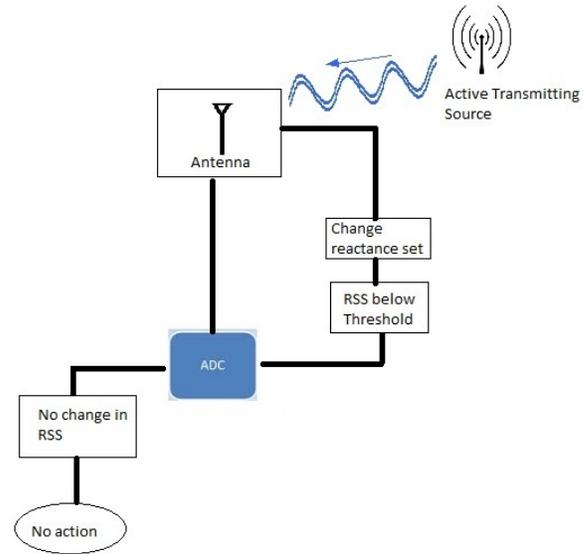


Fig. 6. DOA estimation using power method.

whose strength is measured at receive ESPAR antenna, as soon as received signal strength is dropped from predefined threshold beam is first steered towards right by 1° it is later steered towards left by 2° if received signal strength further drops upon steering beam towards right.

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