

# Comments on "Timing Estimation and Resynchronization for Amplify-and-Forward Communication Systems"

Hani Mehrpouyan, *Member, IEEE*, and Steven D. Blostein, *Senior Member, IEEE*,

**Abstract**—This correspondence first shows that the Cramer-Rao lower bound (CRLB) derivations in the above paper are incorrect. In addition, contrary to the claims in the above paper, the assumptions of perfect timing offset estimation and matched-filtering at the relays affect the generality of the analytical results and cannot be justified assumption.

**Index Terms**—Cooperative communications, amplify-and-forward, synchronization, timing offset estimation, Cramer-Rao lower bound.

## I. INTRODUCTION

In the above-named paper [1], the topic of timing offset and channel estimation in *amplify-and-forward (AF)* relaying cooperative networks consisting of  $K$  relays is analyzed. The authors derive *least squares (LS)* and *maximum-likelihood (ML)* algorithms for the estimation of both channel gains and timing offsets and provide a general expression for the *Cramer-Rao lower bound (CRLB)* [2]. Subsequently, the CRLB is used as a measure to provide insights into the uncertainties of estimated parameters. Finally, a resynchronization algorithm is proposed.

Note that in [1], it is assumed that the timing offsets corresponding to the source-relay links, for  $K$  relays, are perfectly estimated and the received signals at the relays are perfectly matched-filtered.

Notation: italic letters ( $x$ ) are scalars, bold lower case letters ( $\mathbf{x}$ ) are vectors, bold upper case letters ( $\mathbf{X}$ ) are matrices,  $\mathbf{I}$  is the identity matrix, and  $\otimes$ ,  $\Re\{\cdot\}$ ,  $\Im\{\cdot\}$ ,  $(\cdot)^T$ , and  $(\cdot)^H$ , denote Kronecker product, real, imaginary, conjugate, transpose, and conjugate transpose (hermitian), respectively.

**Comment 1: The analysis does not correspond to the actual CRLB but is only an approximation.** In Section III C. of [1] the authors present a derivation for the CRLB

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Hani Mehrpouyan is with the Department of Signals and Systems, Chalmers University of Technology, SE-412 96 Gothenburg, SWEDEN, *phone*: +46(0)704893706, *fax*: +46(0)317721748, and *email*: hani.mehr@ieee.org.

Steven D. Blostein is with the Department of Electrical and Computer Engineering, Queen's University, Kingston, ON K7L3N6 Canada, *phone*: +1(613)533-6561 *fax*: +1(613)533-6615 *email*: steven.blostein@queensu.ca.

for the estimation of timing offsets,  $\epsilon = [\epsilon_1, \epsilon_2, \dots, \epsilon_K]^T$  and channel gains  $\mathbf{h} = [h_1, h_2, \dots, h_K]^T$  and  $\boldsymbol{\xi} = [\xi_1, \xi_2, \dots, \xi_K]^T \triangleq [f_1 h_1, f_2 h_2, \dots, f_K h_K]^T$ . Note that  $\mathbf{h}$  and  $\mathbf{f}$ , the channel gains from relays to destination and sources to relays, respectively, are defined in [1, Eq.(7)], and  $\boldsymbol{\xi}$  is defined in [1, Eq.(9)].

Next, to determine the CRLB the elements of the *Fisher's information matrix (FIM)* according to [1, Eq.(20)] are determined. The expressions

$$\frac{\partial \boldsymbol{\mu}^H}{\partial \Re\{\xi_i\}} = j \frac{\partial \boldsymbol{\mu}^H}{\partial \Im\{\xi_i\}} = (\mathbf{A}_{\epsilon_i} \boldsymbol{\Omega}_i \mathbf{s}_t)^H, \quad \text{and} \quad (1)$$

$$\frac{\partial \boldsymbol{\mu}^H}{\partial \Re\{h_i\}} = j \frac{\partial \boldsymbol{\mu}^H}{\partial \Im\{h_i\}} = (\mathbf{A}_{\epsilon_i} \mathbf{p}_i)^H, \quad (2)$$

are, however, derived incorrectly. Note that in (1) and (2),  $\boldsymbol{\mu}$  is given by [1, Eq.(20)],

$$\boldsymbol{\mu} = \mathbf{A}_\epsilon \mathbf{H} \mathbf{F} \boldsymbol{\Omega} \mathbf{s}_t + \mathbf{A}_\epsilon \mathbf{H} \mathbf{P}, \quad (3)$$

where  $\mathbf{A}_\epsilon \triangleq [\mathbf{A}_{\epsilon_1}, \mathbf{A}_{\epsilon_2}, \dots, \mathbf{A}_{\epsilon_K}]$  and  $\mathbf{s}_t$  are defined in [1, Eq.(6)],  $\boldsymbol{\Omega} = [\boldsymbol{\Omega}_1^H, \boldsymbol{\Omega}_2^H, \dots, \boldsymbol{\Omega}_K^H]^H$  and  $\mathbf{P} = [\mathbf{p}_1, \mathbf{p}_2, \dots, \mathbf{p}_K]$  are defined in [1, Eq.(2)], and  $\mathbf{H} = \text{diag}(h_1, h_2, \dots, h_k) \otimes \mathbf{I}$  and  $\mathbf{F} = \text{diag}(f_1, f_2, \dots, f_k) \otimes \mathbf{I}$  are defined in [1, Eq.(6)].

In deriving (1) and (2) in [1], it is assumed that

$$\frac{\partial h_i}{\partial \Re\{\xi_i\}} = \frac{\partial h_i}{\partial \Im\{\xi_i\}} = \frac{\partial \xi_i}{\partial \Re\{h_i\}} = \frac{\partial \xi_i}{\partial \Im\{h_i\}} = 0. \quad (4)$$

However, note that the terms  $\xi_i$  and  $h_i$ , for  $i = 1, 2, \dots, K$ , are not independent of one another, where  $\xi_i \triangleq f_i h_i$ , for  $i = 1, 2, \dots, K$  [1, Eq. (9)]. Thus, contrary to the results in [1], the terms in (4) should instead be

$$\frac{\partial h_i}{\partial \Re\{\xi_i\}} = -j \frac{\partial h_i}{\partial \Im\{\xi_i\}} = \frac{1}{f_i}, \quad \text{and} \quad (5)$$

$$\frac{\partial \xi_i}{\partial \Re\{h_i\}} = -j \frac{\partial \xi_i}{\partial \Im\{h_i\}} = f_i. \quad (6)$$

More importantly, this shortcoming propagates to the numerical results section, where the approximate CRLB is used to justify the performance of the proposed estimators. The derivation of the true CRLB is beyond the scope of this correspondence.

**Comment 2: The signal model for the received signal at the relays and destination is based on perfect timing synchronization and does not match the AF framework.**

In [1], it is assumed that the timing offsets from source to relays can be estimated perfectly so as align the relay transmissions in the second hop. Unfortunately, this assumption requires absolute timing correction, which is not achieved by the symbol-based synchronizers referenced in [1], unless all delays are short and relay clocks are synchronized. Therefore, the proposed signal model in [1] does not realistically match the AF framework."

Note that due to the timing offset from source to the  $k$ th,  $\epsilon_k^{[sr]}$ , the received training signal model at the  $k$ th relay,  $r_k(t)$ , is given by

$$r_k(t) = f_k \sum_{i=-L_g}^{L_o+L_g+1} s(i)g(t-iT-\epsilon_k^{[sr]}T), \quad (7)$$

where  $g(t)$  is the pulse shaping filter,  $T$  is the symbol duration, and  $s(i)$  is the  $i$ th element of the training sequence transmitted from the source. On the other hand, in [1], due to the assumption of perfect timing offset estimation, the signal model for  $r_k$  is over simplified and the effect of timing offset  $\epsilon_k^{[sr]}$ , for  $k = 1, 2, \dots, R$ , is ignored (see [1, Eq. (2)]). Considering that [1] seeks to address timing synchronization in AF relaying cooperative networks, it can be concluded the the signal model in [1, Eq. (2)] is over simplified, since in practical cooperative communications systems the timing offsets,  $\epsilon_k^{[sr]}$ , for  $k = 1, 2, \dots, R$ , cannot be perfectly estimated and compensated.

The authors of [1] further assume that at the  $k$ th relay, a second training sequence,  $\mathbf{p}_k$ , can be perfectly superimposed on the received signal (see [1, Eq. (3)]). However, this assumption is an over simplification, since in practical communications systems the source and relays are equipped with different oscillators. Therefore,  $\mathbf{p}_k$  and  $\mathbf{s}$  are affected by different timing offsets and subsequently, [1, Eq. (6)] must be rewritten as

$$\begin{aligned} r_k(t) &= \sum_{k=1}^K h_k f_k \bar{\mathbf{A}}_{\bar{\epsilon}_k} \mathbf{W}_k \Omega_k \mathbf{s} + \sum_{k=1}^K h_k \mathbf{A}_{\epsilon_k} \mathbf{W}_k \mathbf{p}_k \\ &+ \sum_{k=1}^K h_k \mathbf{A}_{\epsilon_k} \mathbf{W}_k \mathbf{e}_k + \mathbf{v}, \end{aligned} \quad (8)$$

where  $\bar{\mathbf{A}}_{\bar{\epsilon}_k} \triangleq [\bar{\mathbf{a}}_{-L_g}(\bar{\epsilon}_k), \dots, \bar{\mathbf{a}}_0(\bar{\epsilon}_k), \dots, \bar{\mathbf{a}}_{L_o+L_g-1}(\bar{\epsilon}_k)]$  with  $\bar{\mathbf{a}}_i \triangleq [g(-iT - \bar{\epsilon}_k T), g(-iT + T_s - \bar{\epsilon}_k T), \dots, g(-iT + (L_o Q - 1)T_s - \bar{\epsilon}_k T)]$ ,  $\bar{\epsilon}_k$ , for  $k = 1, 2, \dots, R$ , accounts for the timing offset estimation error at the  $k$ th relay plus the timing offset from the  $k$ th relay to the destination,  $\epsilon_k$ , and the remaining terms in (8) are defined in [1, Eq. (6)]. Based on the training design proposed in [1], the received signal at the destination is affected by two sets of timing offset values  $\bar{\epsilon}_k$  and  $\epsilon_k$ , for  $k = 1, 2, \dots, R$ , instead of only the  $\epsilon_k$  as claimed in [1].

Finally, unlike the results in [1], which assumes that the signal at the relays is perfectly matched-filtered, AF relaying

cooperative communications systems only require the relays to amplify and forward the received signal as shown in prior work in this field [3]–[6]. This is one of the main advantages of AF relaying, which ensures that the relays have a simple structure that can be more easily deployed in practical applications.

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