Electric Circuits

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Lecture 15 (First Order Circuits)
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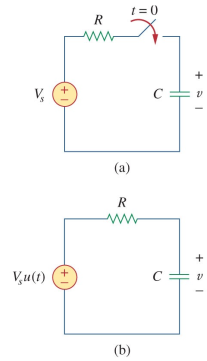
Overview

- This chapter examines RC and LC circuits' reaction to switched sources.
- The circuits are referred to as first order circuits.
- Three special functions, the unit step, unit impulse, and unit ramp function are also introduced.
- Both source free and switched sources are examined.

Step Response of RC Circuit

- When a DC source is suddenly applied to a RC circuit, the source can be modeled as a step function.
- The circuit response is known as the step response.
- Let's consider the circuit shown here.
- We can find the voltage on the capacitor as a function of time.



Step Response of RC Circuit

- We assume an initial voltage of V_0 on the capacitor.
- Applying KCL:

$$\frac{dv}{dt}+\frac{v}{RC}=\frac{V_s}{RC}u(t)$$

- For $t>0$ this becomes:

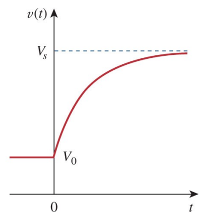
$$\frac{dv}{dt}+\frac{v}{RC}=\frac{V_s}{RC}$$

- Integrating both sides and introducing initial conditions finally yields:

$$v(t)=\begin{cases} V_0, & t<0 \\ V_s+(V_0-V_s)e^{-t/\tau} & t>0 \end{cases}$$

Step Response of RC Circuit

- This is known as the complete response, or total response.
- We can consider the response to be broken into two separate responses:
- The natural response of the capacitor or inductor due to the energy stored in it.
- The second part is the forced response



Forced Response

- The complete response can be written as:
  $v = v_n + v_f$
- Where the nature response is:
  $v_n = V_0 e^{-t/\tau}$
- And the forced response is:
  $v_f = V_s\left(1 - e^{-t/\tau}\right)$
- Note that the eventual response of the circuit is to reach V_s after the natural response decays to zero.

Another Perspective

- Another way to look at the response is to break it up into the transient response and the steady state response:

$$v = v_t + v_{ss}$$

- Where the transient is:

$$v_t = (V_0 - V_s)e^{-t/\tau}$$

- And the steady state is:

$$v_{ss} = V_s$$

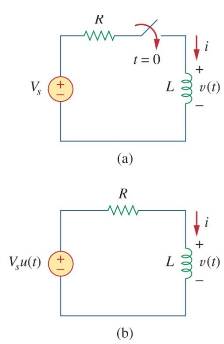
Solved Problem

Solved Problem

Step Response of RL Circuit

- Now we can look at the step response of an RL circuit.
- We will use the transient and steady state response approach.
- We know that the transient response will be an exponential:

$$i_t = Ae^{-t/\tau}$$



Step Response of RL Circuit

- After a sufficiently long time, the current will reach the steady state:

$$i_{ss} = \frac{V_s}{R}$$

- This yields an overall response of:

$$i = Ae^{-t/\tau} + \frac{V_s}{R}$$

- To determine the value of *A* we need to keep in mind that the current cannot change instantaneously.

$$i(0^+) = i(0^-) = I_0$$

Step Response of RL Circuit

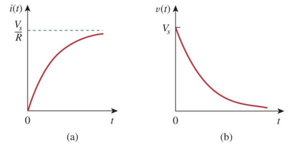
- Thus we can use the *t=0* time to establish *A*

$$A = I_0 - \frac{V_s}{R}$$

- The complete response of the circuit is thus:

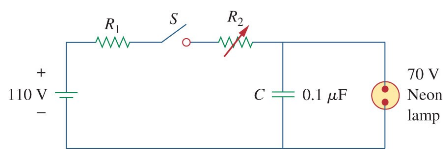
$$i(t) = \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right)e^{-t/\tau}$$

- Without an initial current, the circuit response is shown here.



Application: Delay Circuit

- The RC circuit can be used to delay the turn on of a connected device.
- For example, a neon lamp which only triggers when a voltage exceeds a specific value can be delayed using such a circuit.



Delay Circuit II

- When the switch is closed, the capacitor charges.
- The voltage will rise at a rate determined by:

$\tau = (R_1 + R_2)C$

- Once the voltage reaches 70 volts, the lamp triggers.
- Once on, the lamp has low resistance and discharges the capacitor.
- This shuts off the capacitor and starts the cycle over again.

Solved Problem

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Solved Problem