Electric Circuits

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Lecture 11 (Capacitors and Inductors)
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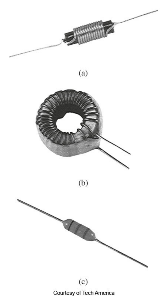
Overview

- This chapter will introduce two new linear circuit elements:
- The capacitor
- The inductor
- Unlike resistors, these elements do not dissipate energy
- They instead store energy
- We will also look at how to analyze them in a circuit

Inductors

- An inductor is a passive element that stores energy in its magnetic field
- They have applications in power supplies, transformers, radios, TVs, radars, and electric motors.
- Any conductor has inductance, but the effect is typically enhanced by coiling the wire up.



Inductors II

- If a current is passed through an inductor, the voltage across it is directly proportional to the time rate of change in current

$$v = L\frac{di}{dt}$$

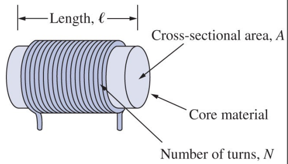
- Where, *L*, is the unit of inductance, measured in Henries, H.
- On Henry is 1 volt-second per ampere.
- The voltage developed tends to oppose a changing flow of current.

Inductors III

- Calculating the inductance depends on the geometry:
- For example, for a solenoid the inductance is:

$$L = \frac{N^2 \mu A}{l}$$

- Where N is the number of turns of the wire around the core of cross sectional area A and length l.
- The material used for the core has a magnetic property called the permeability, μ.



Current in an Inductor

- The current voltage relationship for an inductor is:

$$I = \frac{1}{L}\int_{t_0}^{t} v(\tau)\,d\tau + i(t_0)$$

- The power delivered to the inductor is:

$$p = vi = \left(L\frac{di}{dt}\right)i$$

- The energy stored is:

$$w = \frac{1}{2}Li^2$$

Properties of Inductors

- If the current through an inductor is constant, the voltage across it is zero
- Thus an inductor acts like a short for DC
- The current through an inductor cannot change instantaneously
- If this did happen, the voltage across the inductor would be infinity!
- This is an important consideration if an inductor is to be turned off abruptly; it will produce a high voltage

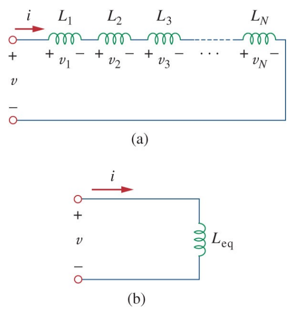
Properties of Inductors II

- Like the ideal capacitor, the ideal inductor does not dissipate energy stored in it.
- Energy stored will be returned to the circuit later
- In reality, inductors do have internal resistance due to the wiring used to make them.
- A real inductor thus has a winding resistance in series with it.
- There is also a small winding capacitance due to the closeness of the windings
- These two characteristics are typically small, though at high frequencies, the capacitance may matter.

Series Inductors

- We now need to extend the series parallel combinations to inductors
- First, let's consider a series combination of inductors
- Applying KVL to the loop:

$$v = v_1 + v_2 + v_3 + \cdots + v_N$$



Series Inductors II

- Factoring in the voltage current relationship

$$v = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + L_3 \frac{di}{dt} + \cdots + L_N \frac{di}{dt}$$
$$= \left(\sum_{k=1}^{N} L_k \right) \frac{di}{dt} = L_{eq} \frac{di}{dt}$$

- Where

$$L_{eq} = L_1 + L_2 + L_3 + \cdots + L_N$$

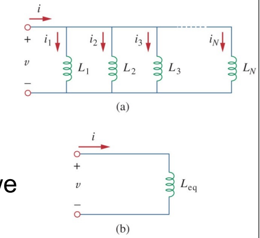
- Here we can see that the inductors have the same behavior as resistors

Parallel Inductors

- Now consider a parallel combination of inductors:
- Applying KCL to the circuit:

$$i = i_1 + i_2 + i_3 + \cdots + i_N$$

- When the current voltage relationship is considered, we have:



$$i = \left(\sum_{k=1}^{N} \frac{1}{L_k}\right)\int_{t_0}^{t} v\,dt + \sum_{k=1}^{N} i_k(t_0) = \frac{1}{L_{eq}}\int_{t_0}^{t} v\,dt + i(t_0)$$

Parallel Inductors II

- The equivalent inductance is thus:

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_N}$$

- Once again, the parallel combination resembles that of resistors

Summary of Capacitors and Inductors

TABLE 6.1

Important characteristics of the basic elements.†

Relation	Resistor (R)	Capacitor (C)	Inductor (L)
v-i:	$v = iR$	$v = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + v(t_0)$	$v = L\frac{di}{dt}$
i-v:	$i = v/R$	$i = C\frac{dv}{dt}$	$i = \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau + i(t_0)$
p or w:	$p = i^2R = \frac{v^2}{R}$	$w = \frac{1}{2}Cv^2$	$w = \frac{1}{2}Li^2$
Series:	$R_{eq} = R_1 + R_2$	$C_{eq} = \frac{C_1C_2}{C_1 + C_2}$	$L_{eq} = L_1 + L_2$
Parallel:	$R_{eq} = \frac{R_1R_2}{R_1 + R_2}$	$C_{eq} = C_1 + C_2$	$L_{eq} = \frac{L_1L_2}{L_1 + L_2}$
At dc:	Same	Open circuit	Short circuit
Circuit variable that cannot change abruptly:	Not applicable	v	i

Applications

- Due to their bulky size, inductors are less frequently used as compared to capacitors, however they have some applications where they are best suited.
- They can be used to create a large amount of current or voltage for a short period of time.
- Along with capacitors, they can be used for frequency discrimination.

Solved Problem

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