Electric Circuits

Hani Mehrpouyan,

Department of Electrical and Computer Engineering,
Boise State University

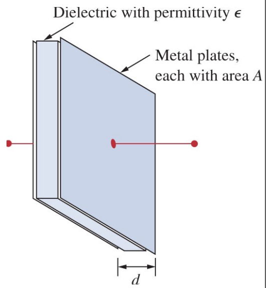
Lecture 10 (Capacitors and Inductors)
Oct 20th, 2015

Overview

- This chapter will introduce two new linear circuit elements:
- The capacitor
- The inductor
- Unlike resistors, these elements do not dissipate energy
- They instead store energy
- We will also look at how to analyze them in a circuit

Capacitors

- A capacitor is a passive element that stores energy in its electric field
- It consists of two conducting plates separated by an insulator (or dielectric)
- The plates are typically aluminum foil
- The dielectric is often air, ceramic, paper, plastic, or mica



Capacitors II

- When a voltage source v is connected to the capacitor, the source deposits a positive charge q on one plate and a negative charge $-q$ on the other.
- The charges will be equal in magnitude
- The amount of charge is proportional to the voltage:

$$q = Cv$$

- Where C is the capacitance

Capacitors III

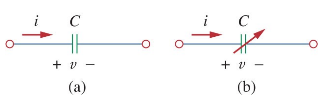
- The unit of capacitance is the Farad (F)
- One Farad is 1 Coulomb/Volt
- Most capacitors are rated in picofarad (pF) and microfarad (μF)
- Capacitance is determined by the geometery of the capacitor:
  - Proportional to the area of the plates (A)
  - Inversely proportional to the space between them (d)

$$C = \frac{\varepsilon A}{d}$$

- ε is the permittivity of the dielectric

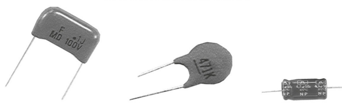
Types of Capacitors

- The most common types of capacitors are film capacitors with polyester, polystyrene, or mica.
- To save space, these are often rolled up before being housed in metal or plastic films
- Electrolytic caps produce a very high capacitance
- Trimmer caps have a range of values that they can be set to
- Variable air caps can be adjusted by turning a shaft attached to a set of moveable plates



Applications for Capacitors

- Capacitors have a wide range of applications, some of which are:
  - Blocking DC
  - Passing AC
  - Shift phase
  - Store energy
  - Suppress noise
  - Start motors



(a) (b) (c)

Courtesy of Tech America

Current Voltage Relationship

- Using the formula for the charge stored in a capacitor, we can find the current voltage relationship
- Take the first derivative with respect to time gives:

$$i = C\frac{dv}{dt}$$

- This assumes the passive sign convention

Stored Charge

- Similarly, the voltage current relationship is:

$$v(t)=\frac{1}{C}\int_{t_0}^{t} i(\tau)\,d\tau + v(t_0)$$

- This shows the capacitor has a memory, which is often exploited in circuits
- The instantaneous power delivered to the capacitor is

$$p = vi = Cv\frac{dv}{dt}$$

- The energy stored in a capacitor is:

$$w=\frac{1}{2}Cv^2$$

Properties of Capacitors

- Ideal capacitors all have these characteristics:
- When the voltage is not changing, the current through the cap is zero.
- This means that with DC applied to the terminals no current will flow.
- Except, the voltage on the capacitor's plates can't change instantaneously.
- An abrupt change in voltage would require an infinite current!
- This means if the voltage on the cap does not equal the applied voltage, charge will flow and the voltage will finally reach the applied voltage.

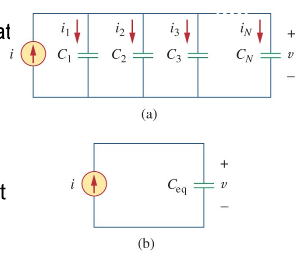
Properties of capacitors II

- An ideal capacitor does not dissipate energy, meaning stored energy may be retrieved later
- A real capacitor has a parallel-model leakage resistance, leading to a slow loss of the stored energy internally
- This resistance is typically very high, on the order of 100 MΩ and thus can be ignored for many circuit applications.

Parallel Capacitors

- We learned with resistors that applying the equivalent series and parallel combinations can simply many circuits.
- Starting with N parallel capacitors, one can note that the voltages on all the caps are the same
- Applying KCL:

$$i = i_1 + i_2 + i_3 + \cdots + i_N$$



Parallel Capacitors II

- Taking into consideration the current voltage relationship of each capacitor:

$$i = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} + C_3 \frac{dv}{dt} + \cdots + C_N \frac{dv}{dt}$$

$$= \left(\sum_{k=1}^{N} C_k \right) \frac{dv}{dt} = C_{eq} \frac{dv}{dt}$$

- Where

$$C_{eq} = C_1 + C_2 + C_3 + \cdots + C_N$$

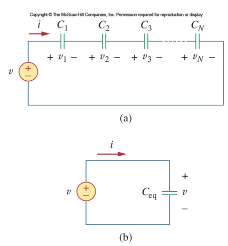
- From this we find that parallel capacitors combine as the sum of all capacitance

Series Capacitors

- Turning our attention to a series arrangement of capacitors:
- Here each capacitor shares the same current
- Applying KVL to the loop:

$$v = v_1 + v_2 + v_3 + \cdots + v_N$$

- Now apply the voltage current relationship



Series Capacitors II

$$v=\frac{1}{C_1}\int_{t_0}^{t} i(\tau)d\tau+v_1(t_0)+\frac{1}{C_2}\int_{t_0}^{t} i(\tau)d\tau+v_2(t_0)+\frac{1}{C_3}\int_{t_0}^{t} i(\tau)d\tau+v_3(t_0)+\cdots+\frac{1}{C_N}\int_{t_0}^{t} i(\tau)d\tau+v_N(t_0)$$

$$=\left(\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\cdots+\frac{1}{C_N}\right)\int_{t_0}^{t} i(\tau)d\tau+v_1(t_0)+v_2(t_0)+v_3(t_0)+\cdots+v_N(t_0)$$

$$=\frac{1}{C_{eq}}\int_{t_0}^{t} i(\tau)d\tau+v(t_0)$$

- Where

$$\frac{1}{C_{eq}}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\cdots+\frac{1}{C_N}$$

- From this we see that the series combination of capacitors resembles the parallel combination of resistors.

Series and Parallel Caps

- Another way to think about the combinations of capacitors is this:
- Combining capacitors in parallel is equivalent to increasing the surface area of the capacitors:
- This would lead to an increased overall capacitance (as is observed)
- A series combination can be seen as increasing the total plate separation
- This would result in a decrease in capacitance (as is observed)

Solved Problem

Boise State University

Solved Problem

Solved Problem

Solved Problem

Solved Problem

Solved Problem