Performance of Circular QAM Constellations with Time Varying Phase Noise

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***Abstract*—Time varying phase noise is a limiting factor in high-speed wireless communication systems, e.g., microwave backhaul links in cellular networks. This paper seeks to investigate the performance of circular *M-ary quadrature amplitude modulations (M-QAMs)* in the presence of time varying phase noise. A new approximate union bound expression for the *symbol-error probability (SEP)* of a specific circular M-QAM constellation is derived. Numerical results show that this expression is accurate at medium-to-high *signal-to-noise ratios (SNRs)* for different constellation orders, M. Next, exact closed-form expressions for the SEP of *binary phase shift keying (BPSK)* and *quadrature phase shift keying (QPSK)* modulations in the presence of phase noise are derived. Extensive simulations are carried out to compare the performance of rectangular and circular QAM modulations in the presence of time varying phase noise, where it is demonstrated that circular QAM modulations can outperform their rectangular counterparts when considering the effect of time varying phase noise.**

I. Introduction

In recent years cellular communications has proven itself as an effective mean of transferring voice as well as high speed data. The demand for high speed wireless links continue to escalate with the evolution of third generation (3G) and fourth generation (4G) cellular networks [1]. However, the higher throughput demands by users cannot be supported without significantly increasing the data rates of the links that interconnect *base-stations (BSs)*, which in turn increases the throughput requirements on the links between *base station controllers (BSCs)* and *mobile switching centers (MSCs)* or the backhaul links. BSC and MSC are interconnected using wired E1/T1 leased lines, optical fiber networks, or high-speed microwave links. Microwave radio backhauls are important for mobile operators since they can be setup much more quickly than wired-line links and due to their cost-effectiveness [2]–[4] decrease the operating and capital costs required to setup a 4G cellular network. As a result, in today's cellular networks more than 50% of mobile backhaul traffic is carried by point to point microwave links [5].

Traditional microwave links employ on-off keying, *binary phase-shift keying (BPSK)*, or differential quadrature phase shift keying. On the other hand, in order to meet the stringent bandwidth efficiencies requirements for these links, the next generation microwave backhaul links are using higher order modulations, e.g., rectangular 16 or 256 *quadrature amplitude modulations (QAMs)*. These modulation are more complex to implement and are prone to phase noise [6], [7]. Circular QAM constellations have been widely employed in satellite broadcasting systems. The interest on circular M-QAM has been due to smaller amplitude fluctuation as compared to rectangular QAM, which results in fewer amplitude levels [8]. However, the potential robustness of circular QAM constellations to phase noise has been overlooked. As a matter of fact, even though high order circular QAM can be effectively used to improve bandwidth efficiency in high speed microwave links, they have not been applied widely in such systems [9], [10]. Therefore, in this paper we seek to demonstrate the potential of circular QAM constellations in improving the bandwidth efficiency and phase noise robustness of high speed wireless links.

Different circular M-QAM constellation designs have been purposed for code division multiple access (CDMA) and orthogonal frequency division multiplexing (OFDM) based communication systems, e.g., [11]–[13]. Moreover, in order to improve the *symbol-error probability (SEP)* of a communication system, in [11]–[14] circular 16-QAM constellations that have been optimized for *additive white Gaussian noise (AWGN)* channels are proposed. However, since the SEP optimizations in [11]–[14] are carried out by ignoring the effect of phase noise, the resulting constellation have an uneven phase distribution amongst its symbols and as shown later in this paper have poor performance in the presence of phase noise.

In [15] the design of circular constellations in the presence of phase offset is investigated and a closed-form SEP for a specific circular constellation is derived. However, the results in [15] are limited to constant phase offset, where it is assumed that phase noise is constant over a set of symbols. In [16], a general expression for the SEP of 2-D modulations affected by both I/Q and phase imbalances are derived. However, the results in [16] are also limited to the constant phase offset scenario and the simulations do not provide any insight on the effect of time varying phase noise on the performance of different QAM constellations. In [17], numerical schemes are used to compare the performance of circular QAM constellations for fiber optic communication systems when considering the effect of time varying phase noise. However, due to the consideration of nonlinear fiber optic channels, the proposed modulation will not perform well in wireless communication systems. The effect of phase noise on *quadrature phase shift keying (QPSK)* modulation has been investigated in [18]. However, in [18], no closed-form

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expression for the SEP of QPSK modulation is provided and the investigation is not expanded to higher order modulations.

In order to improve the SEP performance of a communication system in the presence of time varying phase noise and for ease of demodulation [19], in this paper, we consider the circular QAM constellation in Fig. 1, where each symbol is 90^o apart from its neighboring symbols[1]. Throughout this paper we denote this constellation by *regular circular QAM*. A new approximate and general union bound expression for the SEP of this circular QAM constellation is derived. Next, new closed-form expressions for the SEP of BPSK and QPSK modulations in the presence of time varying phase noise are presented and used to verify the simulation setup in this paper. Finally, the verified simulation setup is used to compare the SEP performance of regular circular QAM in Fig. 1 against that of circular constellations in [11], [12] and the rectangular QAM constellation when considering the effect of time varying phase noise. The contributions of this paper can be summarized as follows:

- A new closed-form approximate union bound expression for the SEP of the regular circular QAM modulation in Fig. 1 is derived. This expression is shown to be very accurate at medium-to-high *signal-to-noise ratios (SNRs)*. Note that even though this result is limited to the constellation in Fig. 1, the approach outlined here provides a framework for obtaining the union-bound expressions on the SEP of other circular QAM constellations.
- New exact and closed-form expressions for the SEP of BPSK and QPSK modulations in the presence of time varying phase noise are derived and verified using numerical results.
- Extensive simulations are carried out to compare the performance of different circular and rectangular QAM constellations in the presence of time varying phase noise. These results provide specific guidelines for designing higher order QAM constellations that are more robust to phase noise. In addition, it is demonstrated that the regular circular QAM in Fig. 1 outperforms rectangular and circular QAM constellations in [11], [12].

The paper is organized as follows: Section II outlines the system model used in this paper. In section III, an approximate and closed-form union bound expression for the SEP of the circular QAM in Fig. 1 is derived and it is validated using simulations for different constellation sizes. In Section IV, the exact closed-form SEP of BPSK and QPSK modulations in the presence of phase noise are derived. In Section V, extensive simulations are carried out to compare the performance of different constellations in the presence of phase noise. Finally, Section VI summarizes the key findings of the paper.

II. System Model

A point-to-point single antenna communication system is considered. Since line-of-sight microwave links are the main focus of this paper, AWGN channels are considered. The equivalent baseband received signal model for the kth symbol can be written as

[1]Note that compared to a rectangular QAM, the symbols in the constellation in Fig. 1 are separated by equal and larger phase angles.

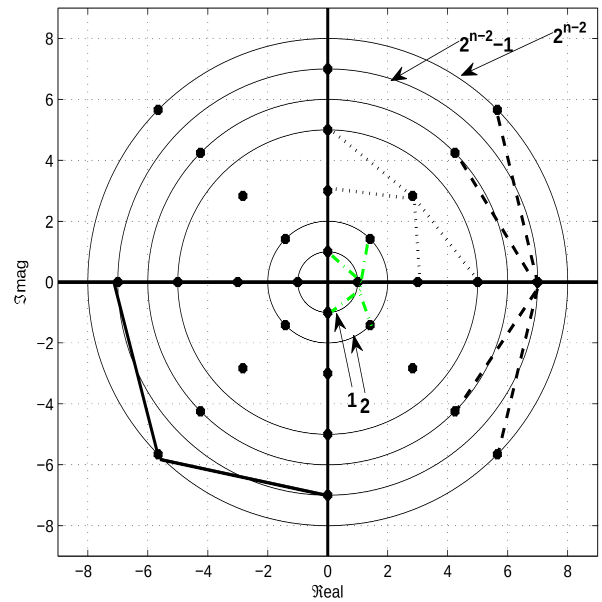

Fig. 1: Schematic illustration of the circular QAM constellation [20].

$$r_k = s_k e^{j\theta_k} + w_k, \tag{1}$$

where s_k is the M-ary modulated signal, where M denotes the constellation size, r_k denotes the received signal, w_k is the zero-mean complex AWGN with variance N_0. We assume that traditional phase noise estimation algorithms, e.g., an extended Kalman filter [21] or a *phase locked loop (PLL)* [19], are applied at the receiver to track the phase noise of free running oscillators. Thus, θ_k in (1) denotes the residual phase noise corresponding to the kth symbol, i.e., θ_k corresponds to the phase tracking error at the receiver. Thus, based on the results in [22], the phase noise process, θ_k, is modeled as a zero-mean Gaussian random variable with variance $\sigma^2_{\theta_k}$. Note that for typical oscillators used in communication receivers the phase noise variance, $\sigma^2_{\theta_k}$, is in the range of 10^{-2} to 10^{-4} rad^2 [23].

III. Derivation of the Union Bound

Since deriving the exact closed-form SEP of the regular circular M-QAM in Fig. 1 is challenging, in this section an approximate closed-form union bound expression for the SEP of this constellation is derived.

In the regular circular M-QAM under consideration, the rings are equally spaced by a distance, a, and each ring has a set of four symbols that are 90° apart from one another. Let us classify the rings as odd and even. Without loss of generality, it is assumed that the odd rings have their symbols positioned on the in-phase, ($\Re eal$), and quadrature-phase, ($\Im mag$), axes, i.e., at 0°, 90°, 180°, and 270°, whereas the symbols on even rings are positioned at an offset of 45°, i.e., 45°, 135°, 225°, and 315°. Let n denote the number of bits in a symbol, i.e., $M = 2^n$, where it is assumed that $n \geq 3$ (note that for $n = 2$ and $n = 1$ the regular circular constellation in Fig. 1 simplifies to that of QPSK and BPSK, respectively, and the SEP for these constellations are available in the literature [19]). Moreover, the total number of rings required to accommodate M symbols is calculated as 2^{n-2} since there are four symbols per ring. Based on Fig. 1 the even and odd rings lie at amplitudes of $A_{\text{even}} = (2^{n-2} - 2m)a$ and $A_{\text{odd}} = (2^{n-2} - 2m - 1)a$, respectively, where $m = 0, \cdots, 2^{n-3} - 1$.

At high SNR values, the AWGN variance, N_0, is small compared to symbol energy, E_s. Therefore, it can be assumed that only the neighboring symbols contribute to erroneous symbol decoding. In Fig. 1 the neighboring symbols for the outer, middle, and inner rings are highlighted. There are three specific categories of symbols that need to be taken into consideration:

- The symbols at the outer most ring (solid line in Fig. 1) have only two neighbors which lie at the $2^{n-2} - 1$th ring.
- Symbols lying on the center rings have four neighbors, these rings can be distinguished into odd rings (dashed line in Fig. 1) and even rings (dotted line in Fig. 1).
- The inner most ring (doted-dashed line in Fig. 1) have four neighbors, two on the same ring and two at immediate outer ring.

In addition, the union bound on symbol error probability, P_e, is given by [19]

$$P_e \leq \frac{1}{M} \sum_{\text{x}=1}^{M} \sum_{\substack{\text{y}=1 \\ \text{x} \neq \text{y}}}^{M} Q \left(\sqrt{\frac{|\text{x}, \text{y}|^2}{2N_0}} \right), \tag{2}$$

where

- $Q(u) = \frac{1}{\sqrt{2\pi}} \int_u^\infty \exp\left(-\frac{v^2}{2}\right) dv$,
- $\text{x} = [x_1, x_2 j]$ and $\text{y} = [y_1, y_2 j]$ denote the symbols, and
- $|\text{x}, \text{y}|$ denotes the Euclidean distance between the symbols x and y, which is given by

$$|\text{x}, \text{y}| = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}\ . \tag{3}$$

Using (2) the union bound on the probability of error, P_e can be determined as

$$P_e < \frac{1}{M} \left(P_{IR}(a) + P_{CR}(a) + P_{OR}(a) \right), \tag{4}$$

where P_{IR}, P_{CR} and P_{OR} are the SEP for inner, center, and outer rings, respectively, and are given in (5a), (5b) and (5c) at the bottom of this page. Note that in (5b) $\beta(i) \triangleq 2^{n-2} - 2i$.

Remark 1: Note that by only taking into account the neighboring symbols, the SEP union bound in (4) is a looser upper bound on the SEP of the regular circular constellation in Fig. 1. However, the numerical results Fig. 2 show that the derived bound is valid for different constellation sizes at medium to high SNRs.

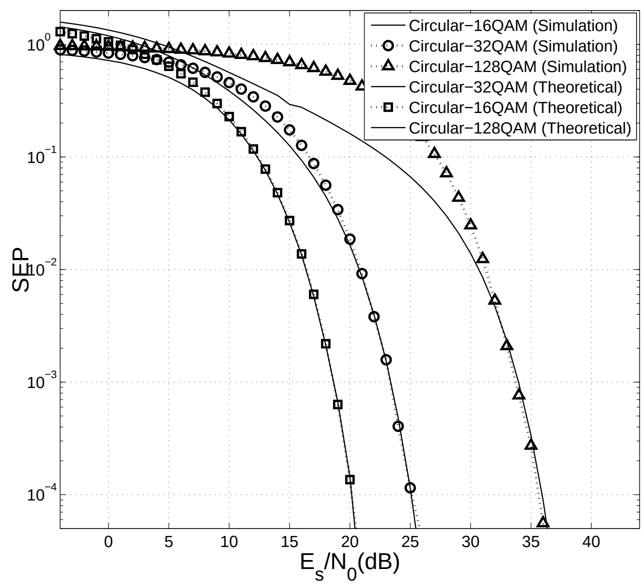

Fig. 2: SEP vs E_s/N_0 in presence of AWGN for $M = [16, 32, 128]$.

IV. Effect of Phase Noise

In this section the closed-form SEP expressions for BPSK and QPSK in the presence of AWGN and phase noise are presented.

$$P_{\text{IR}}(a) = 8Q\left(\sqrt{\left(\frac{\left|[a, 0j], [a\sqrt{2}, a\sqrt{2}j]\right|^2}{2N_0}\right)}\right) + 8Q\left(\sqrt{\frac{\left|[a, 0j], [0, aj]\right|^2}{2N_0}}\right). \tag{5a}$$

$$P_{\text{CR}}(a) = 8 \sum_{i=1}^{2^{n-3}-1} \Bigg\{ Q\left(\sqrt{\frac{\left(\left|[(\beta(i)+1)a, 0j], \left[(\beta(i)+2)\frac{a}{\sqrt{2}}, (\beta(i)+2)\frac{a}{\sqrt{2}}j\right]\right|\right)^2}{2N_0}}\right)$$
$$+ Q\left(\sqrt{\frac{\left(\left|\left[\beta(i)\frac{a}{\sqrt{2}}, \beta(i)\frac{a}{\sqrt{2}}j\right], [(\beta(i)-1)a, 0j]\right|\right)^2}{2N_0}}\right)$$
$$+ 2Q\left(\sqrt{\frac{\left(\left|\left[\beta(i)\frac{a}{\sqrt{2}}, \beta(i)\frac{a}{\sqrt{2}}j\right], [(\beta(i)+1)a, 0j]\right|\right)^2}{2N_0}}\right)\Bigg\}. \tag{5b}$$

$$P_{\text{OR}}(a) = 8Q\left(\sqrt{\frac{\left|\left[2^{n-2}\frac{\sqrt{2}}{2}a, 2^{n-2}\frac{\sqrt{2}}{2}aj\right], [(2^{n-2}-1)a, 0j]\right|^2}{2N_0}}\right). \tag{5c}$$

Next, these closed-form expressions are applied to verify the simulation setup used for investigating the effect of phase noise on M-QAM circular and rectangular modulations.

A. SEP for BPSK

Given that the symbols s_1 or s_2 are transmitted, the conditional *probability distribution function (PDF)* of r_k can be written as

$$p(r|s_1) = \frac{1}{\sqrt{\pi N_0}} e^{-|r-s_1|^2/N_0}, \tag{6}$$

and

$$p(r|s_2) = \frac{1}{\sqrt{\pi N_0}} e^{-|r-s_2|^2/N_0}, \tag{7}$$

respectively. According to (6) and (7), the SEP given that s_1 or s_2 is transmitted can be written as

$$P_e(r|s_1) = \int_{-\infty}^{0} \frac{1}{\sqrt{\pi N_0}} e^{-|r-s_1|^2/N_0} dr \ , \tag{8}$$

and

$$P_e(r|s_2) = \int_{0}^{\infty} \frac{1}{\sqrt{\pi N_0}} e^{-|r-s_2|^2/N_0} dr, \tag{9}$$

respectively. Phase noise rotates the symbol along its axis. Therefore, (8) and (9) can be rewritten in terms of the phase noise parameter, θ_k, as

$$P_e(r|s_1) = \int_{-\infty}^{0} \frac{1}{\sqrt{\pi N_0}} e^{-\left|r-s_1 e^{j\theta}\right|^2/N_0} dr, \tag{10}$$

and

$$P_e(r|s_2) = \int_{0}^{\infty} \frac{1}{\sqrt{\pi N_0}} e^{-\left|r-s_2 e^{j\theta}\right|^2/N_0} dr, \tag{11}$$

respectively. Subsequently, the SEP for BPSK in the presence of phase noise, P_{BPSK_θ}, can be determined as

$$P_{\text{BPSK}_\theta} = \frac{1}{2}\text{erfc}\left(\text{Re}(e^{j\theta})\sqrt{\frac{Es}{N_0}}\right), \tag{12}$$

where

$$\text{erfc}(u) = \frac{2}{\sqrt{\pi}} \int_{u}^{\infty} e^{-u^2} du.$$

$\text{Re}(e^{j\theta})$ denotes the real component of the complex phase noise.

B. SEP for QPSK

A similar approach as above can be used for solving the SEP for the QPSK constellation and the resulting expression is given by

$$\begin{aligned} P_{\text{QPSK}_\theta} =& \frac{1}{2}\text{erfc}\left((\cos\theta - \sin\theta)\sqrt{\frac{E_s}{2N_0}}\right) \\ &+ \frac{1}{2}\text{erfc}\left((\cos\theta + \sin\theta)\sqrt{\frac{E_s}{2N_0}}\right) \\ &+ \frac{1}{4}\text{erfc}\left((\cos\theta + \sin\theta)\sqrt{\frac{E_s}{2N_0}}\right) \\ &\times \text{erfc}\left((\cos\theta - \sin\theta)\sqrt{\frac{E_s}{2N_0}}\right). \end{aligned} \tag{13}$$

Remark 2: The Monte Carlo simulation results in Figs. 3 and 4 verify the closed-form SEP expressions for BPSK and QPSK modulations in (12) and (13), respectively, for different phase noise variances, σ_θ^2. It is shown that the derived closed-form expressions for BPSK and QPSK are valid over a wide range of SNR values and overlap the theoretical results.

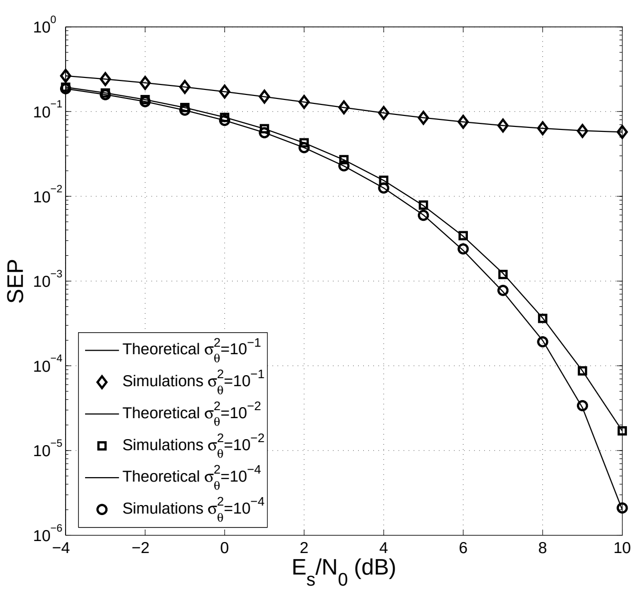

Fig. 3: SEP vs E_s/N_0 for BPSK for phase noise variances $\sigma_\theta^2 = [10^{-2}, 10^{-3}, 10^{-4}]$ rad^2.

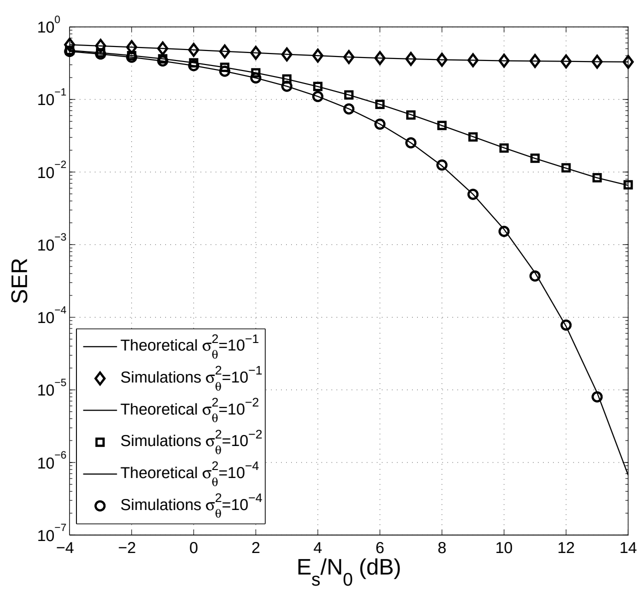

Fig. 4: SEP vs E_s/N_0 for QPSK for phase noise variances $\sigma_\theta^2 = [10^{-2}, 10^{-3}, 10^{-4}]$ rad^2.

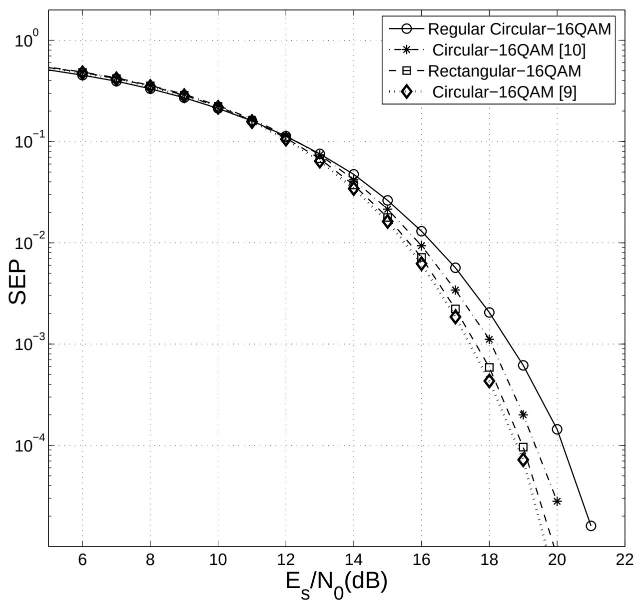

Fig. 5: Comparison of the SEP performance of regular circular 16QAM, the circular 16QAMs in [11], [12], and rectangular 16QAM when ignoring the effect of phase noise.

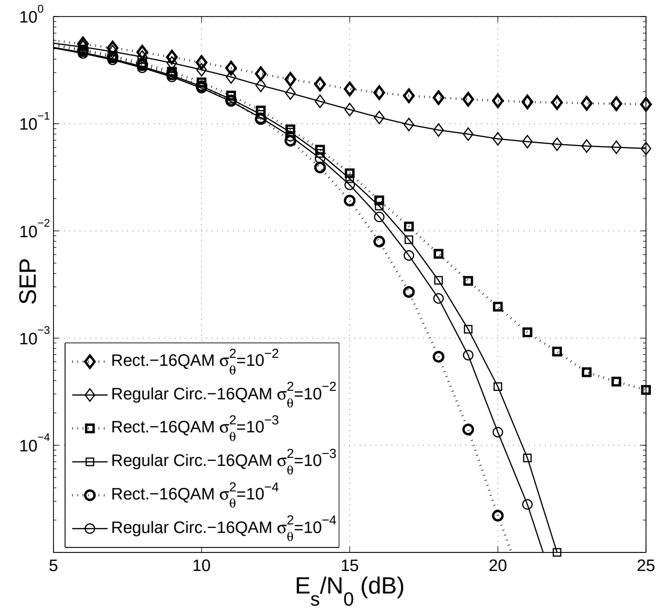

Fig. 6: SEP of regular circular 16QAM vs. rectangular 16QAM in the presence of phase noise.

V. Numerical and Simulation Results

Throughout this section Monte-Carlo simulations are used to investigate the SEP performance of regular circular 16-QAM, the circular 16-QAM in [11], the circular-16QAM in [12], and rectangular 16-QAM constellations in the presence of phase noise.

Fig. 5 depicts the SEP performance of the regular circular 16-QAM and that of [11], [12], and rectangular 16-QAM when ignoring the effect of phase noise. It is observed that at an SEP of 10^{-5}, the constellations in [11], [12], and rectangular-QAM outperform the regular circular 16-QAM outlined in this paper by 2 dB. This is anticipated, since the circular constellations in [11], [12] are optimized for AWGN channels while ignoring the effect of phase noise. Therefore, as shown in 5, they outperform the regular circular 16-QAM used in this paper. However, due to this optimization and the fewer number of rings and amplitude levels, each ring within the constellations in [11], [12] has a larger number of symbols, which in turn reduces the phase gap between symbols. In comparison, the regular circular 16-QAM depicted in Fig. 1 has wider and equal phase error free regions for each symbol (each symbol is $90°$ apart from adjacent symbols). Furthermore, it can be noted that in the case of rectangular 16-QAM the symbols on the center rectangles have a smaller phase difference compared to the most inner and outer rings. As a result, it is expected that they would significantly contribute to the SEP in the presence of phase noise.

In Figs. 6, 7, and 8 phase noise is also introduced to the communication system. Monte Carlo simulation is performed on aforementioned modulation schemes for different phase noise variances. Fig. 6, 7, and 8 show that phase noise appears to decrease the dynamic range of the SEP, since it rotates the symbols and reduces the overall error free region for each symbol, in turn resulting in higher SEPs for all the modulations under consideration. It is also observed that the regular circular 16QAM constellation outlined in this paper, performs 4 dB better than the rectangular 16QAM at intermediate phase noise variances, i.e., $\sigma_\theta^2 = [10^{-2}, 10^{-3}]$ rad^2. In addition, note that at an SEP of 10^{-4} and phase noise variance of $\sigma_\theta^2 = 10^{-3}$ rad^2, the overall SEP for a communication system employing rectangular 16QAM does not improve by increasing the SNR whereas the regular circular 16QAM in Fig. 1 continues to perform better as the SNR increases. This demonstrates the potential of the regular circular QAM constellation outlined in this paper in improving the performance of communication systems when phase noise is a dominating factor, e.g., point-to-point high-speed microwave links [9].

Figs. 7, and 8 also show that regular circular 16QAM also outperforms the circular 16QAM constellations in [11] and [12] at intermediate phase noise rates. This can be anticipated since these constellations have been designed by ignoring the effect of phase noise. This result demonstrates the importance of considering the effect of AWGN and other impairments such as phase noise when designing a modulation for high-speed communication systems. Finally, we would like to point out that the regular circular QAM constellation in this paper can be altered (by changing the number of symbols on the outer or inner rings) to outperform the rectangular QAM for any specific phase noise variance through an exhaustive search. However, this is the subject our current research and beyond the scope of this paper.

VI. Conclusion

In this paper, a closed-form expression for the SEP union bound of the regular circular M-QAM in Fig. 1 is derived. Numerical results demonstrated that the derived SEP bound is accurate at medium-to-high SNRs. Next, the exact closed-form SEP expressions of BPSK and QPSK in the presence of

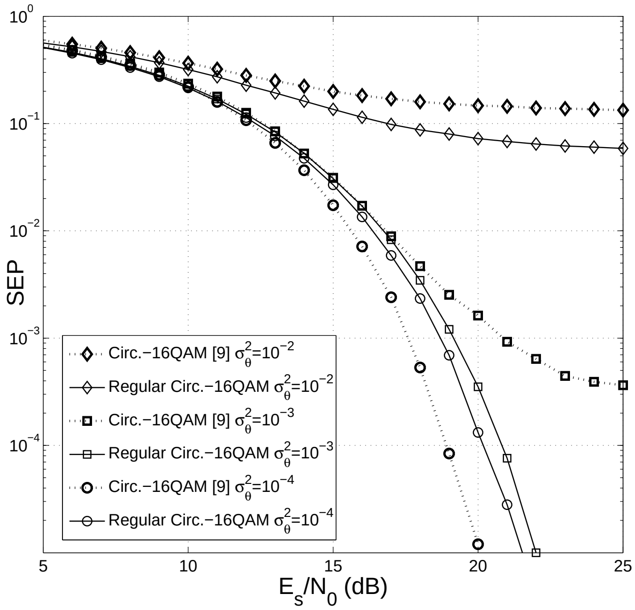

Fig. 7: SEP of regular circular 16QAM vs. circular 16QAM in [11] in the presence of phase noise.

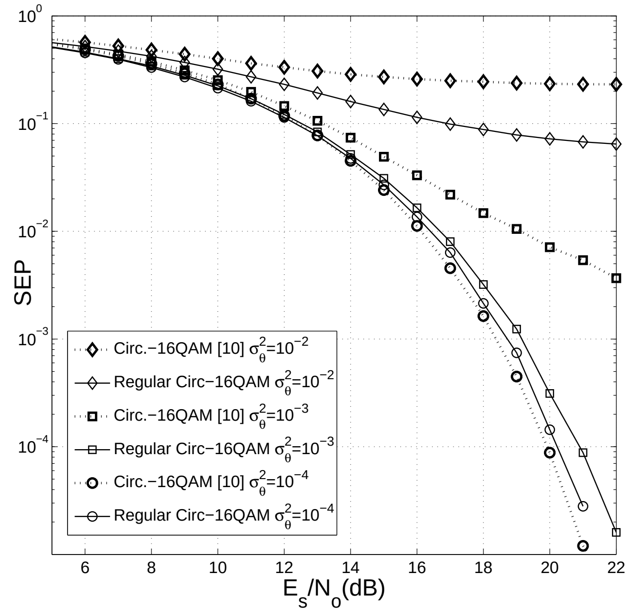

Fig. 8: SEP of regular circular 16QAM vs. circular 16QAM in [12] in the presence of phase noise.

phase noise are derived and these expressions are shown to be accurately determining the SEP of these constellations over a wide range of SNRs and phase noise variances. Extensive simulations demonstrate that the considered regular circular 16-QAM outperforms existing circular and rectangular 16-QAM constellations at intermediate phase noise variances by an average of 4 dB. Note that this constellation can be altered to outperform the rectangular M-QAM constellations for any specific phase noise variance and SNR through an exhaustive search. However, finding a systematic and low complexity method for determining such circular constellations is subject of current research.

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